Time-Dependent Network Pricing and Bandwidth Trading

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Outline

- Time-dependent Network Access and Pricing
  - Bandwidth-on-demand
- A selfish routing model with user preferences
  - Nash Equilibrium (NE) and Social Optimum (SO)
- Social planner: Using pricing to achieve SO
- Service provider (ISP): Revenue Maximization & Efficiency Loss
  - Full information: Differentiated prices over time and users
  - Partial information: Differentiated prices over time
- Conclusion and extensions

Time-dependent Network Access and Pricing

- Service provider (ISP) owns a network
- Each user picks some time to access the network
  - Example: time-of-the-day pricing, Bandwidth-on-demand
  - Users have different preference on access time.
  - Users' decisions based on time-preference, congestion, price
- Questions
  - Model?
  - Is Nash Equilibrium efficient?
  - How to price the users to achieve efficient use of the network?
  - How to maximize the revenue of service provider (ISP)? Does it cause efficiency loss?

Selfish routing model with user preferences

- Divide a day's time into *n* time slots.

6am	8am	10am	12pm	2pm	4pm	6pm	8pm

  - Warmer colors means (typical) busier times…
  - Slots are analogous to parallel links

- Assume a large number of users, divided into *K* classes [1], with the number of users in class *k*

$$d_k = \sum_{i=1}^{n} d_k^i$$

- "Payoff" for a class-*k* user which chooses slot *i* is

$$v_k^i = u_k^i - l_i(x_i) \qquad x_i := \sum_{j=1}^{K} d_j^i$$

  - u_k^i is the "utility" of the user; $l_i(.)$ is the strictly increasing congestion delay; x_i is the number of users in slot *i*

[1] Srinivas Shakkottai, Eitan Altman, Anurag Kumar, "Multihoming of users to Access Points in WLANs: A population game perspective", JSAC 2007

Selfish routing model with user preferences

- If prices are charged, then the user's payoff becomes

$$f_k^i := u_k^i - l_i(x_i) - p_k^i$$

- Social welfare

$$V(\mathbf{d}) \;=\; \sum_{i=1}^{N}\sum_{k=1}^{K}[u_k^i - l_i(x_i)] \cdot d_k^i$$

Nash Equilibrium and Social Optimum

- Condition of NE: For each class *k*

$$\begin{cases} u_k^i - p_k^i - l_i(x_i) = \lambda_k & \forall d_k^i > 0 \\ u_k^i - p_k^i - l_i(x_i) \leq \lambda_k & \forall d_k^i = 0 \end{cases}$$

- NE exists: It is a solution of the strictly convex optimization problem

$$\max_{\mathbf{d}} \quad \sum_{i=1}^{N}\sum_{k=1}^{K}(u_k^i - p_k^i)d_k^i - \sum_{i=1}^{N}\int_0^{x_i} l_i(y)dy$$

$$st \quad \sum_{i=1}^{N} d_k^i \leq d_k, \forall k; d_k^i \geq 0, \forall k, i$$

- Condition of Social optimum (SO)

$$\begin{cases} \frac{\partial V(\mathbf{d})}{\partial d_k^i} = u_k^i - l_i(x_i) - x_i \cdot l_i'(x_i) = \beta_k & \forall d_k^i > 0 \\ \frac{\partial V(\mathbf{d})}{\partial d_k^i} = u_k^i - l_i(x_i) - x_i \cdot l_i'(x_i) \leq \beta_k & \forall d_k^i = 0 \end{cases}$$

Social planner: Achieving SO by pricing

- NE the same as SO, if the price is

$$p_k^i = x_i \cdot l_i'(x_i)$$

- Simple pricing scheme*:* independent of users' classes, utilities u_k^i
- However, the price values at the equilibrium are related to u_k^i
- Not surprising since the price is the "externality" caused by a class-*k* user in slot *i*

ISP: Revenue Maximization (with Full Information)

- Idealized assumption: ISP has full knowledge of the users' classes and utilities u_k^i
- Given an allocation $\{d_k^i\}$, the ISP can charge the user surplus

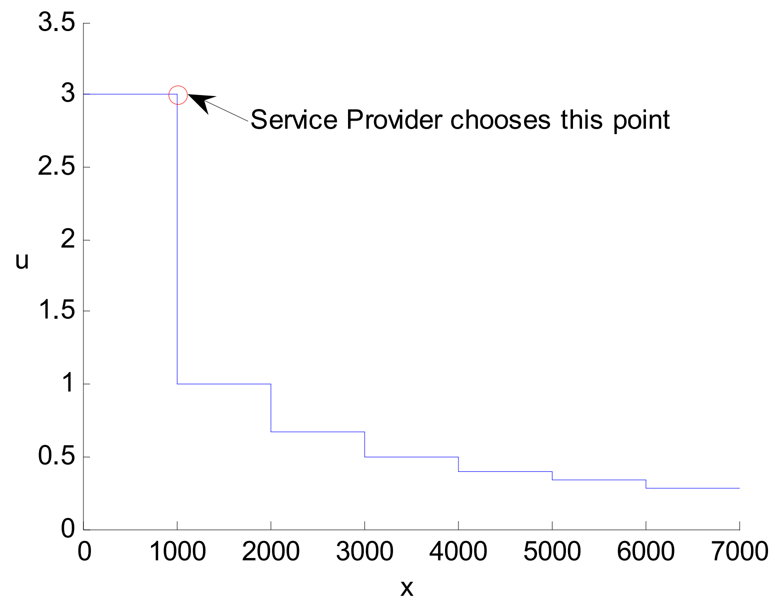
$$u_k^i - l_i(x_i)$$

from a class-k user in slot i

- So, maximizing its revenue is the same as maximizing the social welfare
- (Find allocation that maximizes SW; compute and post the resulting prices; there is a unique NE for users and it maximizes SW.)
- ISP has full advantage since he obtains all the social welfare

ISP: Revenue Maximization (with Partial Information)

- If the SP can not differentiate the user classes, then MR -- the equilibrium that maximizes ISP revenues -- is not SO
- Use $\rho = V_{MR}/V_{SO}$ to measure the Price of Anarchy
- An example where the ratio is close to 0
  - One time slot, $l^1(.)=0$
  - Class 1,2,3,…, each with population 1000
  - $u_1=3$;
  - $u_i=2/i;, i=2,3,\ldots$
  - $V_{MR}=3000$, V_{SO}=infinity
- Very different from the full-information case where V_{MR} = infinity



1000 users have utility 3;
1000 have utility 1; …
There is one time slot, l(.) = 0.
Max x.p s.t. x are users with u > p
Is achieved for p = 3.

A continuum model of Partial Information

- Since there is a single price in each time slot, the NE solves

$$\max_{\mathbf{d}} \quad \sum_{i=1}^{N}\sum_{k=1}^{K} u_k^i d_k^i - \sum_{i=1}^{N} p^i x_i - \sum_{i=1}^{N}\int_0^{x_i} l_i(y)dy$$

$$\text{st} \quad \sum_{i=1}^{N} d_k^i \le d_k, \forall k; d_k^i \ge 0, \forall k, i$$

- It is equivalent to

$$\max_{x_i, i=1,\ldots,N} \quad U(x_1, x_2, \ldots, x_N) - \sum_{i=1}^{N} p^i x_i - \sum_{i=1}^{N}\int_0^{x_i} l_i(y)dy$$

  - $U(x_1, x_2, \ldots, x_N)$ is the maximal total utility of the users, given the number of users in each slot
  - Then the population becomes a single user with utility function *U(.)*

A continuum model (continued)

- U is concave in d^i, i=1,2,…,N
- A continuum of users, whose utility vector

  $\mathbf{u} = (u_1, u_2, \ldots, u_N)$ follows some distribution $f(\mathbf{u})$
- Reasonable to model U as a smooth, increasing, concave function.
- Relation between $\nabla U(\mathbf{x})$ (marginal utility) and $\mathbf{x}$:
  - Any user in slot i must satisfy (with his utility vector $\boldsymbol{u}$)

    $u_i \geq \bar{u}_i$ and $u_j - \bar{u}_j \leq u_i - \bar{u}_i, \forall j \neq i.$ where $\bar{u}_j$ is the marginal utility in slot j *(because* $\bar{u}_i$ *= disutility of i)*
  - $$x_i = \int_{\bar{u}_i}^{\infty} f_i(u_i) Pr\{u_j \leq u_i - \bar{u}_i + \bar{u}_j, \forall j \neq i \,|\, u_i\} du_i$$

Revenue Maximization v.s. Social Optimum

- The ISP solves (MR)

$$\begin{array}{ll} \max_{\mathbf{p},\mathbf{x}} & \mathbf{p}^T\mathbf{x} \\ \text{st} & \nabla U(\mathbf{x}) = \mathbf{p} + \mathbf{l}(\mathbf{x}) \end{array}$$

where the i's element of $\mathbf{l}(\mathbf{x})$ is $l^i(x_i)$

- The solution satisfies (if ***x***$_M$ >0)

$$\nabla U(\mathbf{x}_M) + \nabla^2 U(\mathbf{x}_M) \cdot \mathbf{x}_M = \mathbf{q}(\mathbf{x}_M)$$

where the i's element of $\mathbf{q}(\mathbf{x})$ is $x_i \cdot l_i'(x_i) + l_i(x_i)$

- At Social Optimum,

$$\text{maximize } V(\mathbf{x}) := U(\mathbf{x}) - \sum_{i=1}^{N} x_i \cdot l^i(x_i)$$

$$\nabla U(\mathbf{x}_S) = \mathbf{q}(\mathbf{x}_S)$$

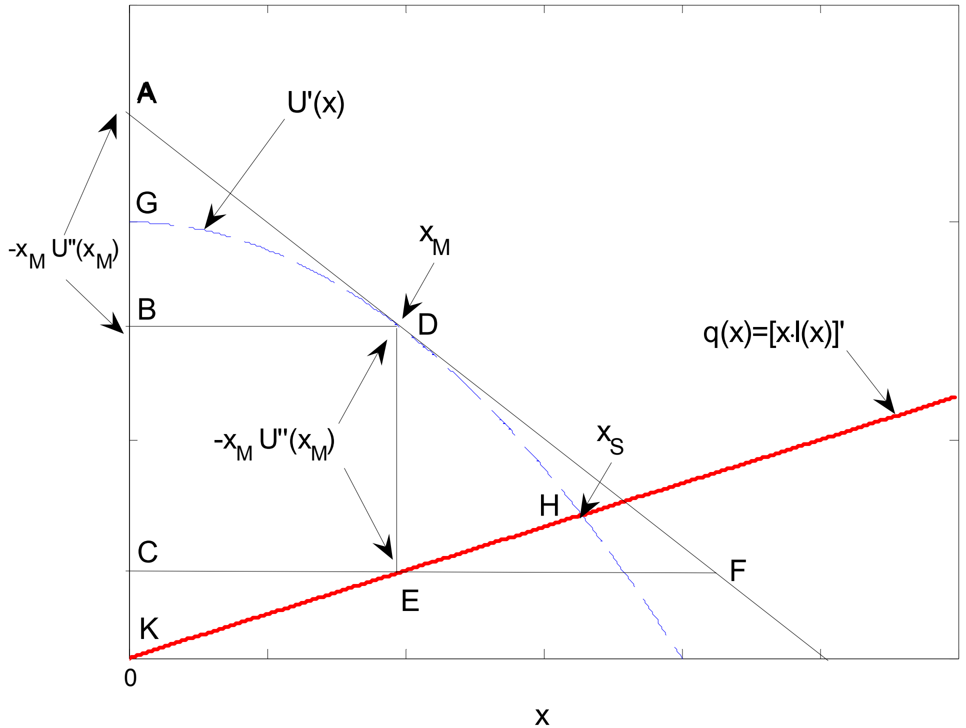
Efficiency loss

$\rho = V_{MR} / V_{SO} \geq 2/3$, under either set of the following conditions:

(a) Every element of $\nabla U(\mathbf{x})$ is concave in $\mathbf{x}$, and $\mathbf{x}_S \geq \mathbf{x}_M$ element-wise;

(b) $(\mathbf{x}_S - \mathbf{x}_M)^T \nabla U(\mathbf{x})$ is concave in $\mathbf{x}$ (at least for $\mathbf{x}$ along the line from $\mathbf{x}_M$ to $\mathbf{x}_S$). Note that condition (a) implies (b) and therefore is stronger.

Intuitively, these conditions require that low-utility users are not dominant



One-dimension case

Example

- Uniform utility distribution

$$f(\mathbf{u}) = 1, \forall \mathbf{u} \in [0,1]^N$$

- And assume the delay is 0
- By symmetry, at SO and MR, all slots have the same number of users.
- Relation between the marginal utility and *x*

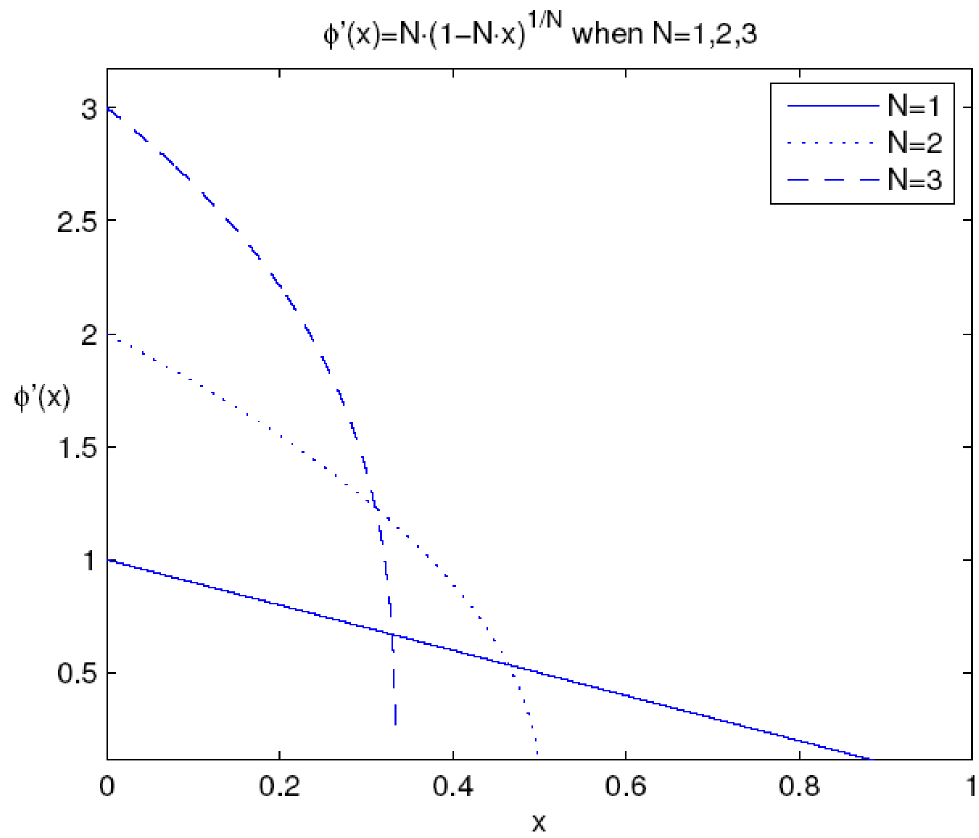
$$\begin{aligned} x &= x_i = \int_{\bar{u}}^{1} 1 \cdot Pr\{u_j \le u_i, \forall j \ne i \,|\, u_i\} du_i \\ &= \int_{\bar{u}}^{1} 1 \cdot u_i^{N-1} du_i \\ &= \frac{1-\bar{u}^N}{N} \end{aligned}$$

Then

$$\bar{u} = (1 - N \cdot x)^{1/N}$$

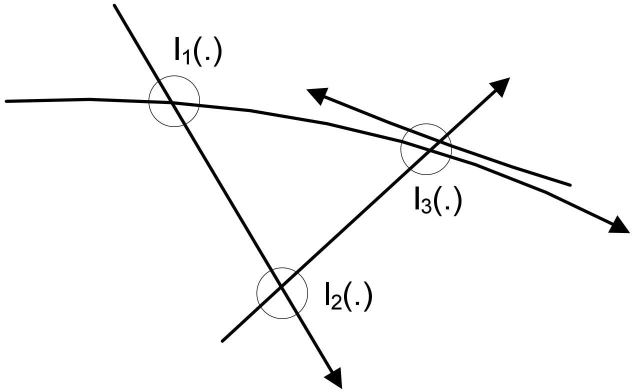
Example

- Then SO and MR can be computed
- The concave assumption is satisfied, so the POA should be not worse than 2/3
- In this example, POA is at least 3/4 (achieved when N=1)



Extensions and Future works

- A network of links instead of a single link. (2-dimensional routing: time and space)
- Most conclusions still hold
  - Social planner: if each link charges the marginal cost, social optimum is achieved.
  - Service provider
    - With full information, maximizing revenue=maximizing social welfare
    - With partial information, there is an efficiency loss, although the worst-case ratio may differ quantitatively



Extensions and Future works

- Users have elastic traffic, use multiple time slots, with difference service types
  - Utility function of user *i*

    $u_i(\mathrm{s}_i)$, where $\mathrm{s}_i = (\mathrm{s}_i^1, \mathrm{s}_i^2, \ldots, \mathrm{s}_i^N) \in \mathcal{R}^N$
- Experimental study
  - Relate the model to empirical data, typical time-preferences of a population

Thank you!