An Efficient Mechanism for Network Bandwidth Auction

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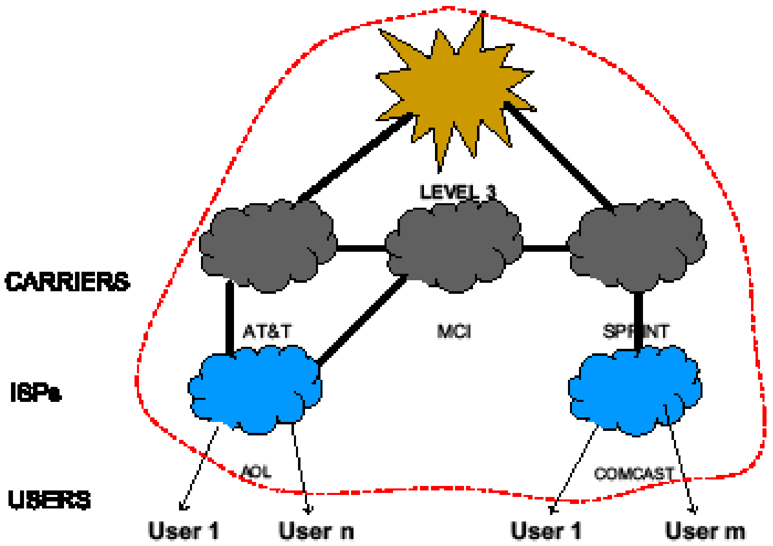
Workshop on Bandwidth on Demand 2008
April 11, 2008
Salvador da Bahia, Brazil

Outline

1. **Motivating Resource Allocation Problem**
2. The NSP Auction Mechanism
3. Equilibrium Properties
4. The Double-sided Version for Resource Exchange

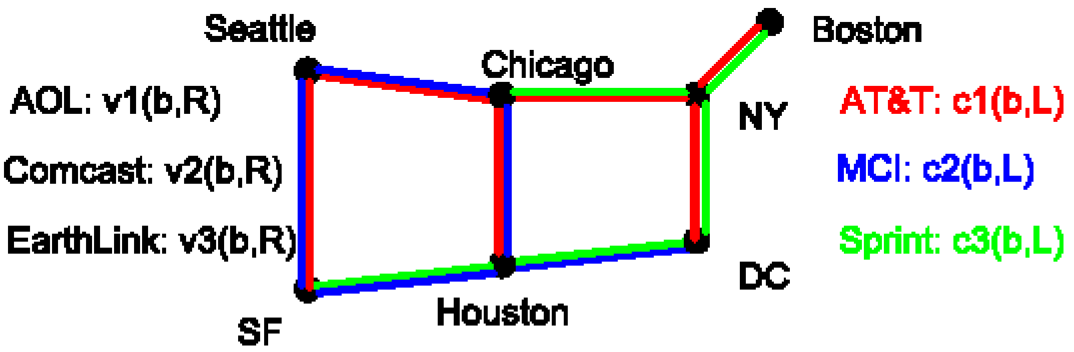
Internet Structure



- ★ *Domains:* Owned and operated by independent entities – have own information and are selfish
- ★ *Players:* Users, ISPs, Carriers
- ★ *Question?* How can we devise mechanisms such that these entities acting selfishly end up achieving the system objective

Decentralized Network Resource Exchange

★ Resource Allocation at the Network level:

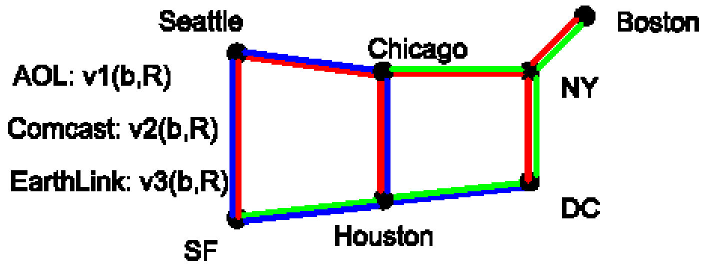


★ Network operator has private cost $c_j(y; l)$ on link l

★ Service provider has private utility $v_i(x;R_i)$ for bundle R_i

★ <u>System Objective:</u> maximize *Social Welfare* $= \Sigma_i\, v_i(x_i) - \Sigma_i\, c_j(y_j)$

Network Resource Allocation Among Strategic Players



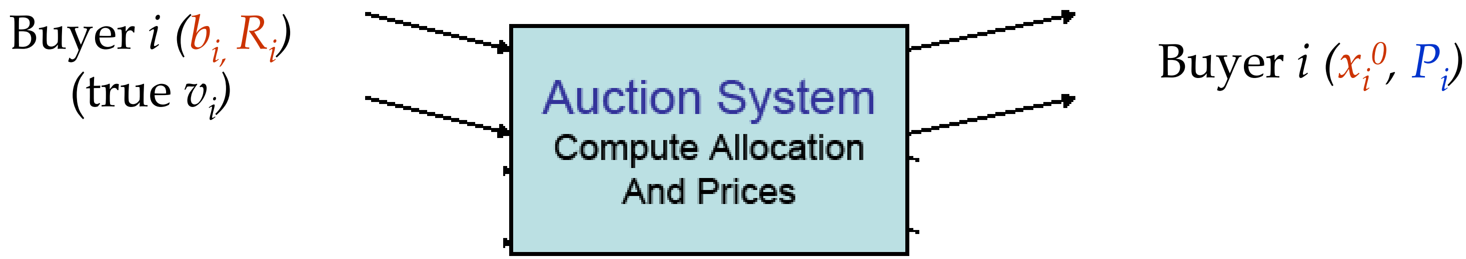
- ★ L divisible goods $1,\dots,L$; C_l units of good l
- ★ n buyers & *the Auctioneer*
- ★ Buyer i wants bundle R_i, has payoff function

$$u_i(x_i,w_i) = v_i(x_i) - w_i$$

  - Assume: v_i is strictly increasing, concave and twice differentiable

- ★ Social Welfare: $S(x) = \sum_i v_i(x_i)$
- ★ <u>SYSTEM OBJ</u>: x^{**} that *max* $S(x)$ sub. to capacity constraints
  - Capacity constraints: $\sum_{i2\ P(i)} x_i \cdot C_l\ 8\ l$
    - $P(l)$ = set of Buyers who have l in their bundle

The Auction System



- ★ Buyer i reports a bid signal b_i for bundle R_i
- ★ BUYER OBJ: To pick b_i to maximize

$$u_i(b_i,b_{-i}) = v_i(x_i(b_i,b_{-i})) - P_i(b_i,b_{-i})$$

- ★ SYSTEM and BUYER Objectives need to be aligned
  - By picking the right allocation function x^0 and payment functions P_i

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The Network Second Price Mechanism

- ★ Buyers submit bids $b_i=(\beta_i,d_i)$: willing to pay $\$\beta_i$ per unit up to d_i units of bundle R_i

- ★ Mechanism maximizes "social welfare" sub. to capacity and demand constraints

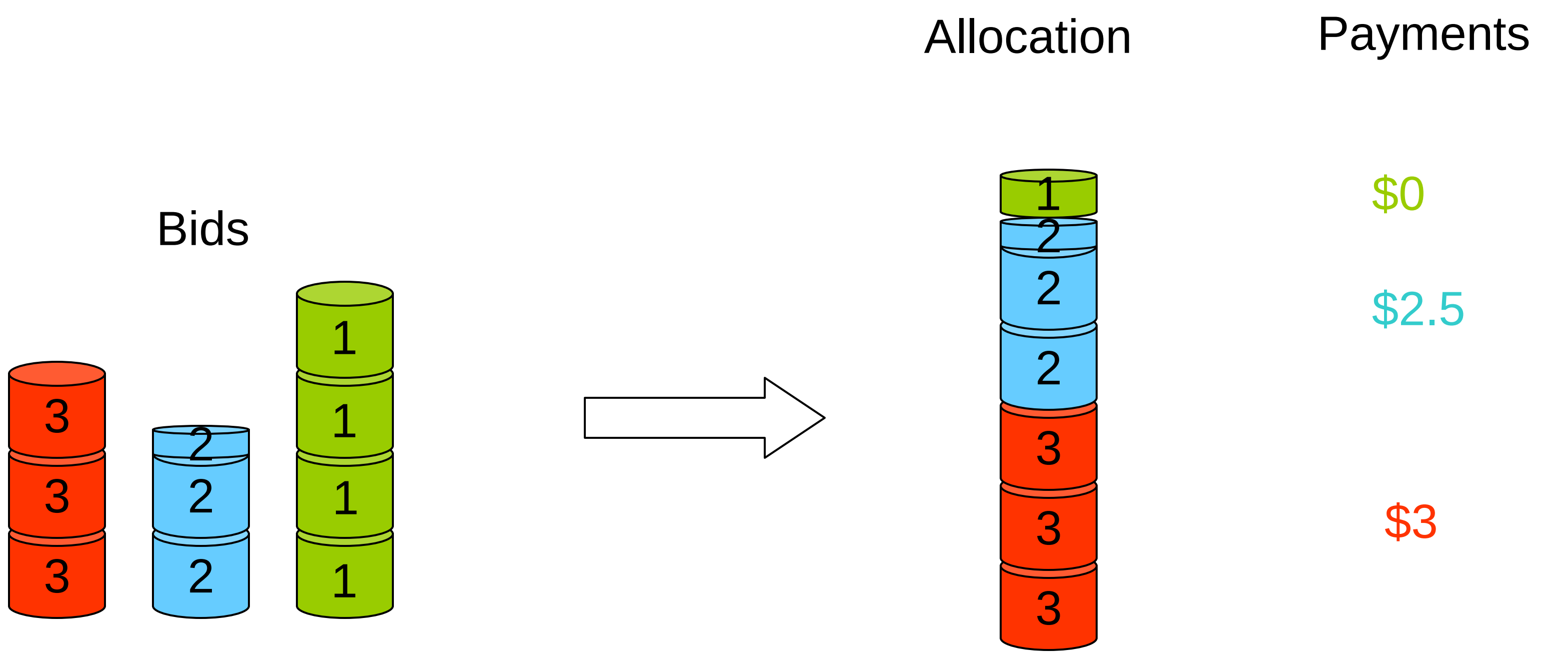
$$x^0\ 2\ arg\ max\ \Sigma_i\ \beta_i\ x_i$$
$$s.t.\quad \Sigma_{i\ 2\ P(l)}\ x_i \cdot C_l,\ 8\ l\ 2\ [0:L]$$
$$x_i\ 2\ [0,d_i],\ 8\ i=1,\ldots,n$$

- ★ Buyer i receives x_i^0 of R_i and pays $P_i(b_i,b_{-i}) = \sum_{j \neq i} \beta_j\ [x^0_j(-i) - x^0_j\ ]$
  - $x^0_j(-i)$ is auction allocation when bidder i does not participate

An Illustrative Example

★ **Example:** C=6 of one good among Three Players,

- Player R bids ($3/unit, 3 units), Player B ($2/unit, 2.5 units) & Player G ($1/unit, 4 units)



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Nash Equilibrium Analysis

★ The payoff of buyer i is

- $u_i(b_i, b_{-i}) = v_i(x^0_i(b)) - P_i(b)$

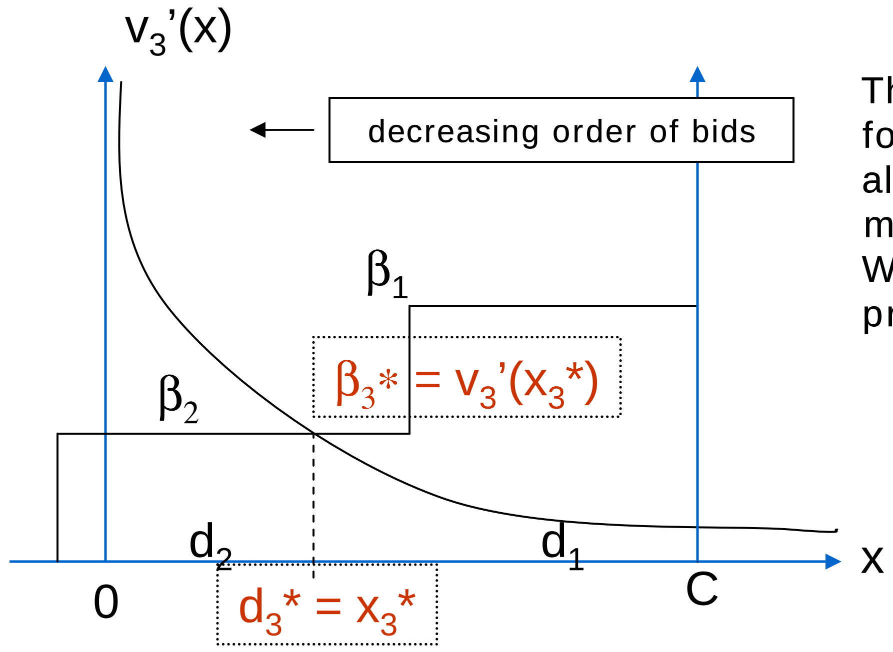
★ A *Nash equilibrium* is a bid profile $b^* = (b_1^*, \ldots, b_n^*)$ such that

$$b_i^* \;\; 2 \; arg\; max_{b_i} \; u_i(b_i, b_{-i}^*), \quad 8 \; i$$

★ *A* Nash equilibrium (allocation x^*) is efficient if $\sum_i v_i(x_i^*) = S_{max} = \sum_i v_i(x_i^{**})$

Theorem: There exists a Nash Equilibrium b^* with efficient allocation $x^* = x^{**}$

Proof Idea



The Lagrange multipliers for the auction optimization also turn out to be Lagrange multipliers for the Social Welfare maximization problem

★ Inefficiency and Reserve Prices

- Many Nash equilibria, not all efficient but some inefficient Nash equilibria can be eliminated through reserve prices: Each winner pays a reserve price plus usual payment

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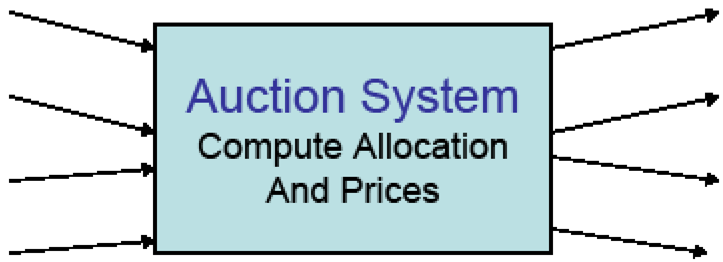
The Double-sided NSP Mechanism

★ L divisible goods, n buyers, m sellers (& *The Auctioneer*)

Buyer i $(b_i = (\beta_i, d_i), R_i)$
(true v_i)

Seller j $(a_j = (\alpha_j, s_j), l_j)$
(true c_j)



Buyer i (x_i^0, P_i)

Seller j $(y_j^0, -T_j)$

★ Buyer i wants a bundle R_i, Seller j offer only one good l_j
★ Social Welfare: $S(x,y) = \sum_i v_i(x_i) - \sum_j c_j(y_j)$
★ "Capacity" constraints: $\sum_{i \in B(l)} x_i \leq \sum_{j \in S(l)} y_j, \forall l$

★ <u>SYSTEM OBJ</u>: (x^{**}, y^{**}) that *max* $S(x,y)$ s.t. capacity constraint satisfied
★ <u>BUYER/SELLER OBJ</u>: Pick bids β_i, α_j to maximize their net payoffs

Theorem: There exists a Nash Equilibrium (b*, a*) with efficient exchange (x*,y*) = (x**,y**)

Related Work

- ★ Semret and Lazar (1999) Progressive Second Price (PSP) Mechanism for a single good and only buyers
  - Maille-Tuffin (2004) provided a multi-bid generalization for single good
  - Courcoubetis, et al. (2004, 2005) suggested a PSP generalization to the network case but mechanism has efficiency problems
- ★ Johari-Tsitsiklis (2005) proposed a very general VCG-like single-sided mechanism with 1-dimensional bids and "nice" bidding functions
  - Yang-Hajek (2005, 2006) also proposed 1-dimensional bid function VCG-like mechanisms, special case
  - Maheswaran-Basar (2004) proposed a similar mechanism ESPA
- ★ Stoenescu-Ledyard (2006) give another network auction mechanism

Summary

★ We have proposed a VCG-like mechanism for both single-sided *and* double-sided auctions

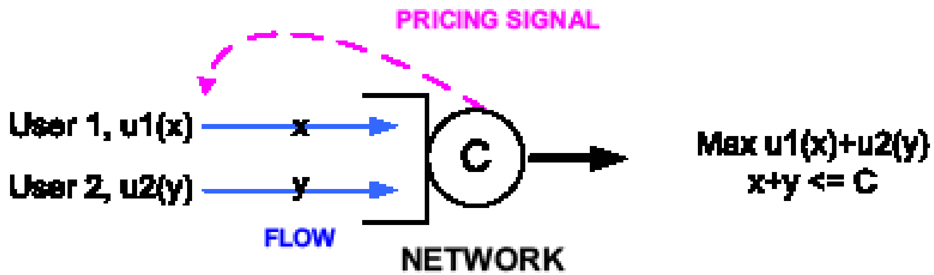
- Generalization of Semret and Lazar (1999) to the network case as well as double-sided case
- Two-dimensional bids and more “natural” bidding functions
- Generalizes to multiple routes between a source-destination pair

★ Showed existence of an efficient Nash equilibria

- Some inefficient Nash equilibria can be mitigated through reserve pricing
- Provided characterization of efficient Nash equilibria

Flow Control and Pricing

★ Resource Allocation at (Fine-grained) User-level:



★ Pricing signals can be used to decompose *Flow Optimization* problem [Kelly'97]

- Primal-Dual decomposition: Users solve *Primal* problem, Network solves *Dual* problem

★ Fundamental understanding of TCP: Internet instability unlikely even with heterogeneous TCP usage [Low'99, Walrand'00, La'04, etc.]