UNIVERSITY OF PALERMO

American Options Based Service Pricing For Virtual Operators

TTI Lab

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- ✓ Scenario
- ✓ Resource/Price Model
- ✓ American Call Option Pricing Formulation
- ✓ Numerical results

Introduction

Two broad categories of service providers

- Network Operators

  Who own their infrastructures
  Who sustain fixed costs (CAPEX)
  Who sustain recurrent costs (OPEX)

- Virtual Operators

  Who do not own their infrastructures
  Who hire network resources
  Who sustain recurrent costs (OPEX)

Introduction

Both operators provide the same service with the same QoS, but they have different objectives

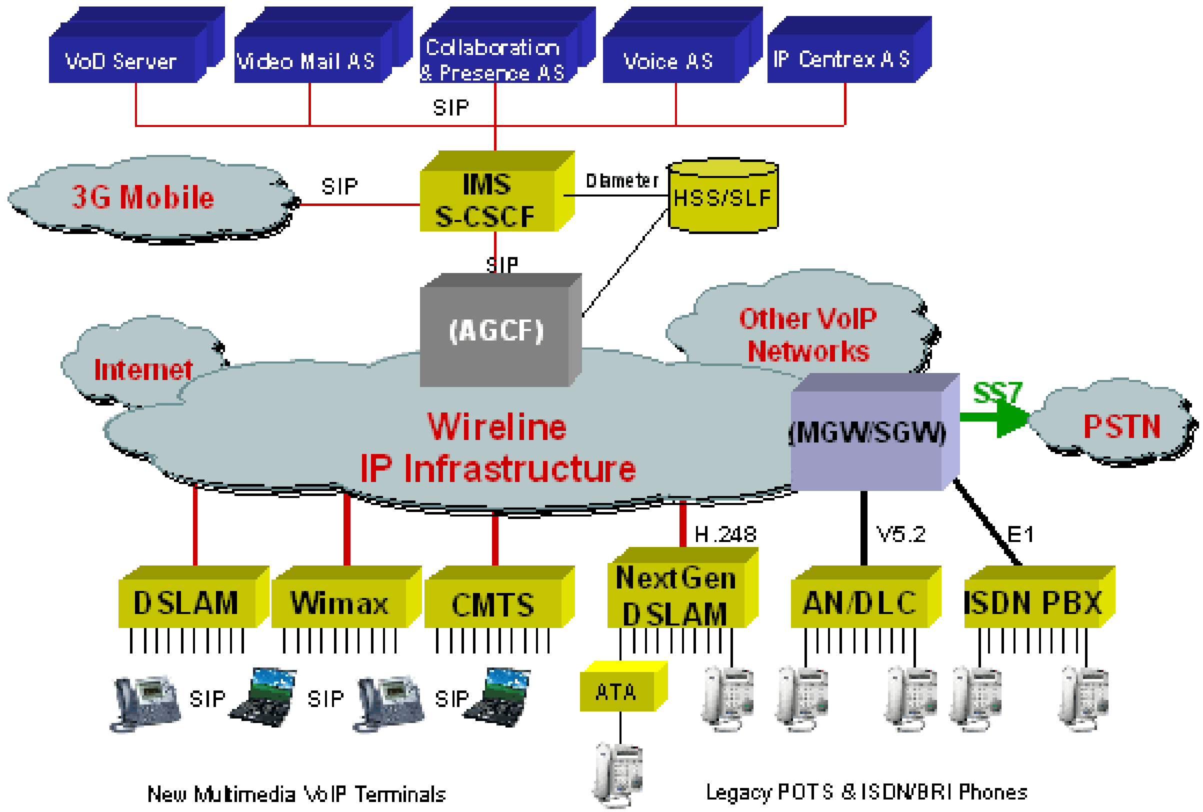
- Network Operators want to recover their costs at a limited risk with traditional pricing schemes (flat or time based)
- Virtual Operators want to stay in the market, trying to attract as many users as they can over their virtual network through new pricing schemes (time varying, congestion pricing)

In this scenario there are two main problems

- Co-existence in the market of two "identical" services (at least from users' perspective)
- Avoid arbitrage opportunities

Scenario

The adoption of new telecommunication technologies increase service alternatives for end users



Example an user can make a call by PSTN, UMTS or INTERNET Network

Scenario

In this context, QoS differences between two services vanish passing from a network platform to another

What really differentiates services is how operator has decided to stay in the market, because its economic objectives reflect in the pricing strategy

EXAMPLE

A network operator (N.O.) sustain OPEX & CAPEX	A virtual operator (V.O.) sustain OPEX only
N.O. want to recover all his cost and it is risk averse	V.O. wants to stay in the market and it is a risky player
N.O. provides services at a flat rate	V.O. may provide a service with evolutionary pricing which encourage the use of resources
N.O. sell minutes of traffic at predictable price	V.O. sell minutes of traffic at time-varying price following a demand-offer law

Scenario

As said above, a problem is to avoid arbitrage opportunities

But what is an arbitrage?

An arbitrage is an opportunity to buy an asset at lower price in a market and to resell the same asset at a higher price in another market free risk.

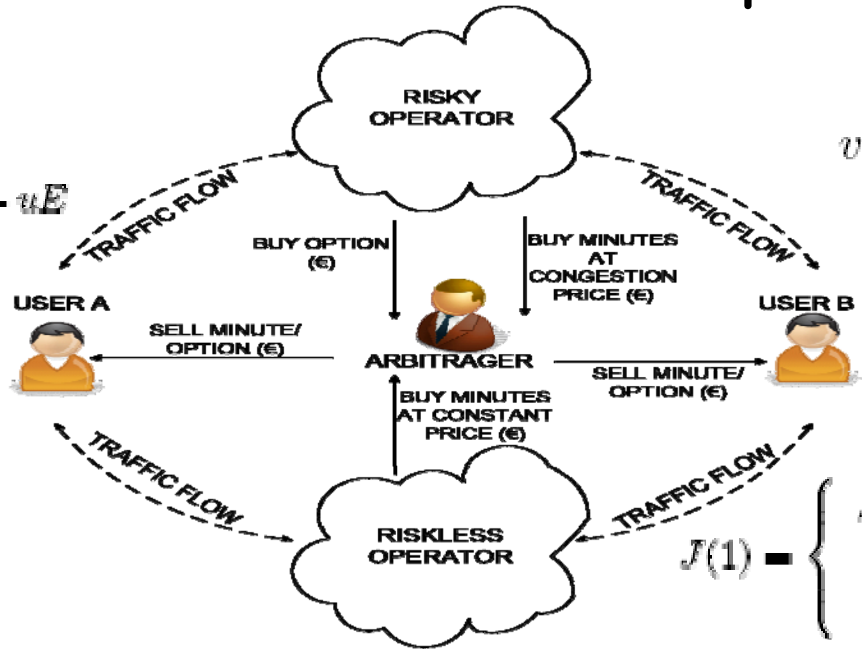
Telecommunication example

$S^+ > K$

$$\begin{cases} \text{portfolio1:} & (S^+ - K) + uE \\ \text{portfolio2:} & vS^+ \end{cases}$$

$S^- < K$

$$\begin{cases} \text{portfolio1:} & uE \\ \text{portfolio2:} & vS^- \end{cases}$$



To prevent arbitrages

$$v = \frac{S^+ - K}{S^+ - S^-} \; ; \; u = \frac{S^-}{E} \frac{S^+ - K}{S^+ - S^-}$$

$$J(1) = \begin{cases} S^+ - K & \text{with probability } \frac{S - S^-}{S^+ - S^-} \\ 0 & \text{with probability } \frac{S^+ - S}{S^+ - S^-} \end{cases}$$

Resource/Price Model

Our main assumption on single link

- The risk averse provider N.O. has a *CAC* based link represented by M/M/N*/N* queue, with traffic intensity (Λ/μ) and blocking probability ε_1
- The risky provider V.O. has a virtual link of capacity N*, with a M/M/∞ queuing discipline, traffic intensity (Λ/μ) and congestion probability ε_2 (capacity N* can be surpassed with probability ε_2)

The probability threshold ε_2 can be engineered to give users the same perceived quality as the blocking probability ε_1

We use a diffusion approximation for the stochastic process of the state of the queue M/M/∞, with i_t the number of users in the queue at t

$$di_t = -\mu\left(i_t - \frac{\lambda}{\mu}\right)dt + \sqrt{2\lambda}dW_t$$

Mean reverting Ornstein-Uhlenbeck process

Resource/Price Model

A fair congestion pricing is to set the price in proportion to the probability of i_t exceeding the threshold N*. A simplified formulation

$$p_t = \psi(i_t) = \begin{cases} \gamma \dfrac{1 - \Phi\left(\frac{N^* - \frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}}\right)}{1 - \Phi\left(\frac{i_t - \frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}}\right)} & i_t \leq N^* \\ \gamma & i_t > N^* \end{cases}$$

Where Φ is a standard gaussian distribution function and γ is a normalization parameter that can be calculated with numerical method by equating the revenues for risky and risk averse operators

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T i_t \psi(i_t) dt = \frac{\lambda}{\mu} p$$

Where p is the €cent/minute rate given by riskless operator

American Call Option Pricing Formulation

In order to develop an option pricing formulation the risk neutralized price must be calculated

$$\hat{p}_t = \hat{\psi}(i_t) = \psi(i_t - \frac{\delta\lambda}{\mu})$$

Where δ is the "amount of neutralization" which can be calculated with numerical method by

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T i_t \psi(i_t - \frac{\delta\lambda}{\mu}) dt = \alpha(1+\beta)\frac{\lambda}{\mu} p$$

Where α is the ratio between the costs and the revenues of the virtual operator and β is the riskless interest rate
In other words, the second term is the gain of an investor, should he invest $\alpha(\lambda/\mu)p$ €cent/min in a riskless asset

American Call Option Pricing Formulation

The American option price for the virtual risky operator is

$$J_0(T, i_0, K) = \max_{\tau \in (0,T]} E^*[max(\hat{p}_\tau - K, 0)|i_0]$$

$J_0(T,i_0,K)$ has the dimensions of €cent/min and it is a bonus charge that allows a user to purchase traffic at K (strike price) rather then at p_t any time that $p_t > K$ …

… But this fact could not be perceived as fair by users (he cannot control the real time evolution of the price)

So two different pricing alternatives formulations are given

American Call Option Pricing Formulation

Formulation [1]
valid on a call by call basis, depending on the declared service duration T and the initial state i_0

$$O_0(T, i_0, K) =$$
$$J_0(T, i_0, K)E\{\int_0^T I[(p_t > K)|i_0]dt\}$$

Formulation [2]
valid if a distribution is available for duration T and the initial state i_0

$$O_0(K) = \int\int O_0(T, i_0, K)f(T)g(i_0)dT di_0$$

Where f(T) and g(i_0) are the probability density functions of T and i_0

O_0(..) is a response charge paid from the user, in €

Numerical Result

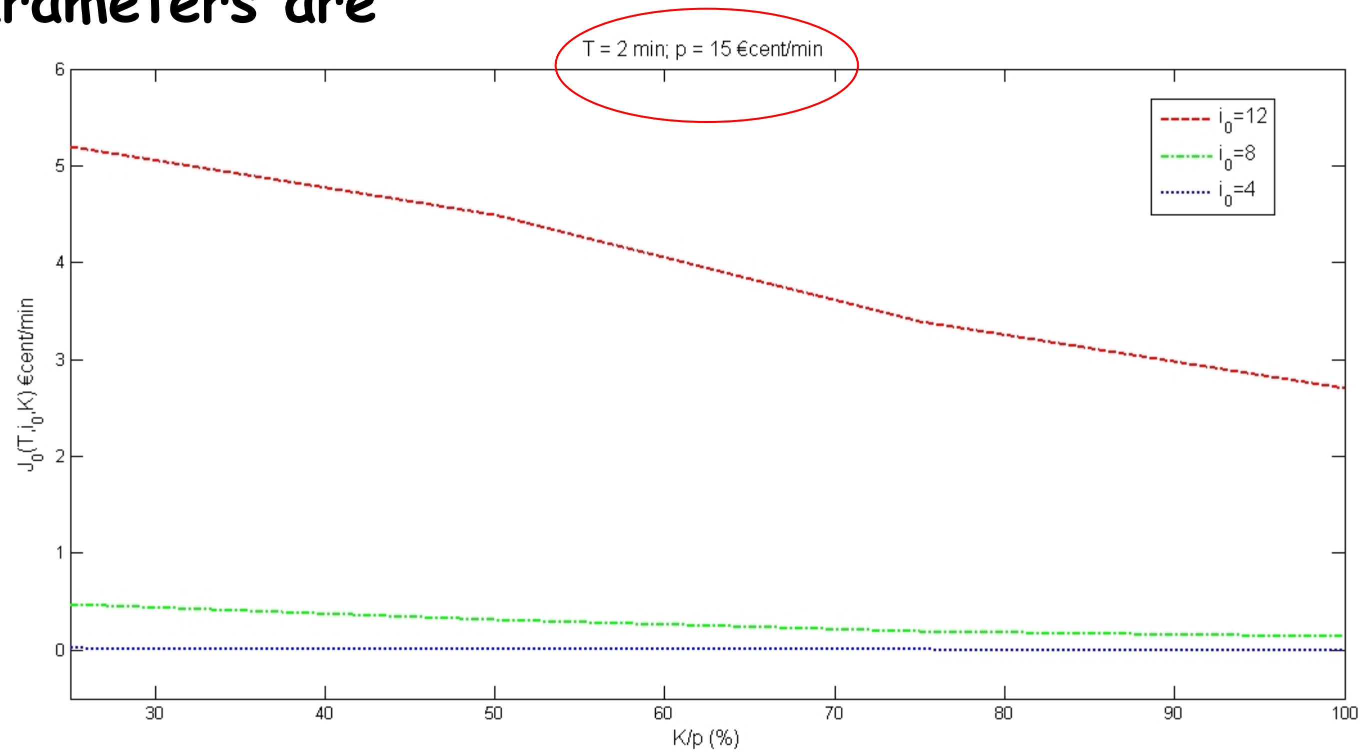
We Monte Carlo simulations have been developed to reproduce the Ornstein-Uhlenbeck process of the spot price under both natural and risk neutralized distribution with parameters

- **mean arrival rate λ=0.05[1/sec],**
- **mean departure rate μ=0.0055 [1/sec],**
- **cost/revenues ratio α=75%,**
- **riskless interest rate β=10%,**
- **QoS threshold N*=17,**
- **reference price of network operator p=15 [€cent/min]**

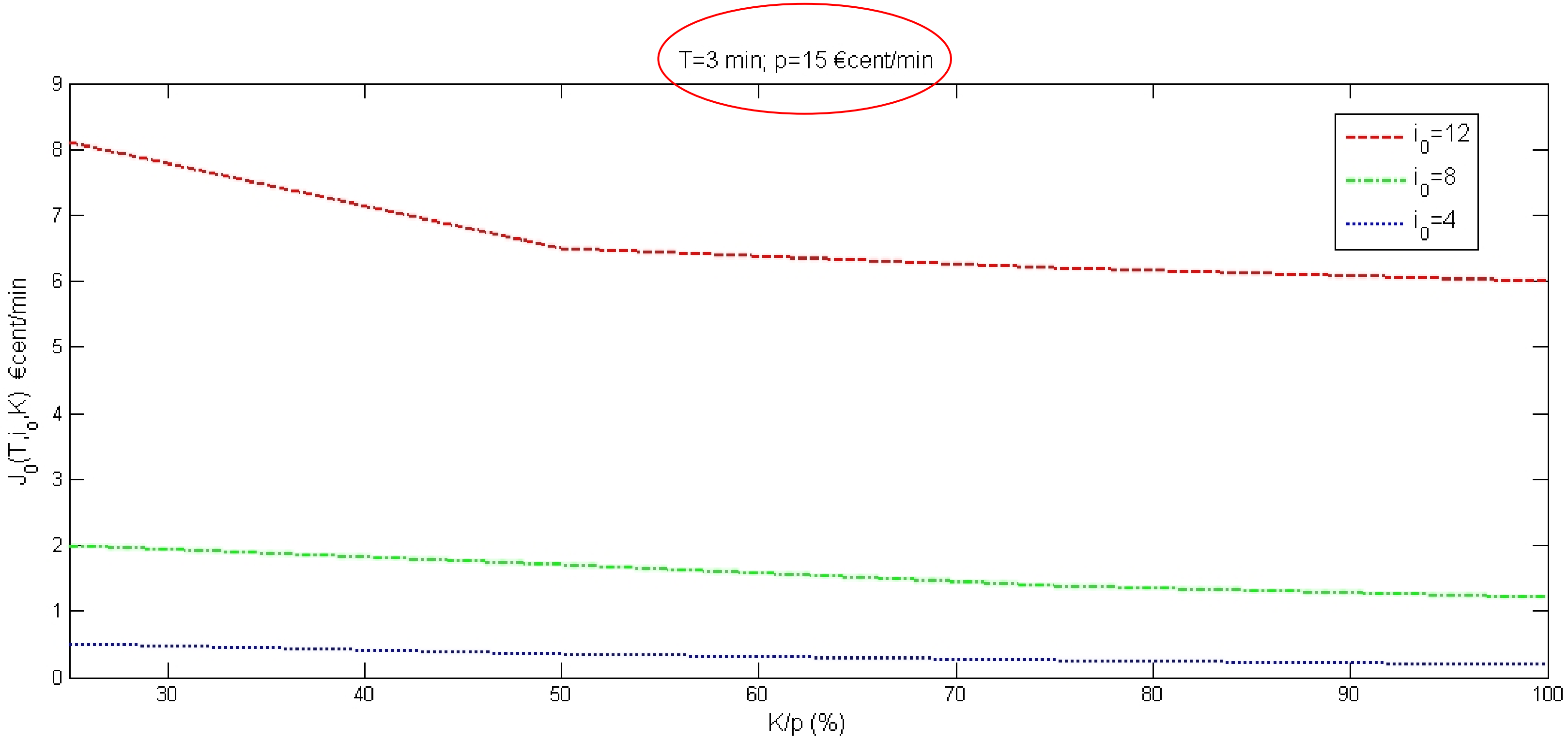
Numerical Result

The estimated values of parameters γ and δ are γ =282.3 δ =3.95%

The estimated option values $J(T, i_0, K)$ and response charges $O_0(K)$ parameters are



Numerical Result



Conclusion

With option pricing scheme the risk balance between low network utilization and high rates is shared between users and operators, giving rise to very simple tariff plans based on response charges and a maximum rate per minute

Our American scheme optimizes the usage of resources, in fact, when the network is lightly loaded it encourages new arrivals and discourages new arrivals during heavy-loaded periods

The proposed method is very easy to understand by users (after all, it is a simple response charge!)

American option pricing can become computationally too heavy but variance reduction techniques (Cross Entropy) can be a viable solution (to be further investigated by the authors)

Thanks

Questions?