A heuristic approach to revenue maximisation in a competitive bandwidth-on-demand wireless market

Fernando Beltran
Matthias Roggendorf

THE UNIVERSITY OF AUCKLAND BUSINESS SCHOOL

PING Research group

Information Systems and Operations Management Department
University of Auckland Business School

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Agenda

- Problem formulation
  - Background and related literature
  - Assumptions taken
- A model of competition
  - Complete information
  - Bayesian model
- Simulation approach
  - Heuristic Algorithm for revenue maximisation
  - Simulation results
- Conclusions

The research problem: Providers may directly compete for customers on the access level

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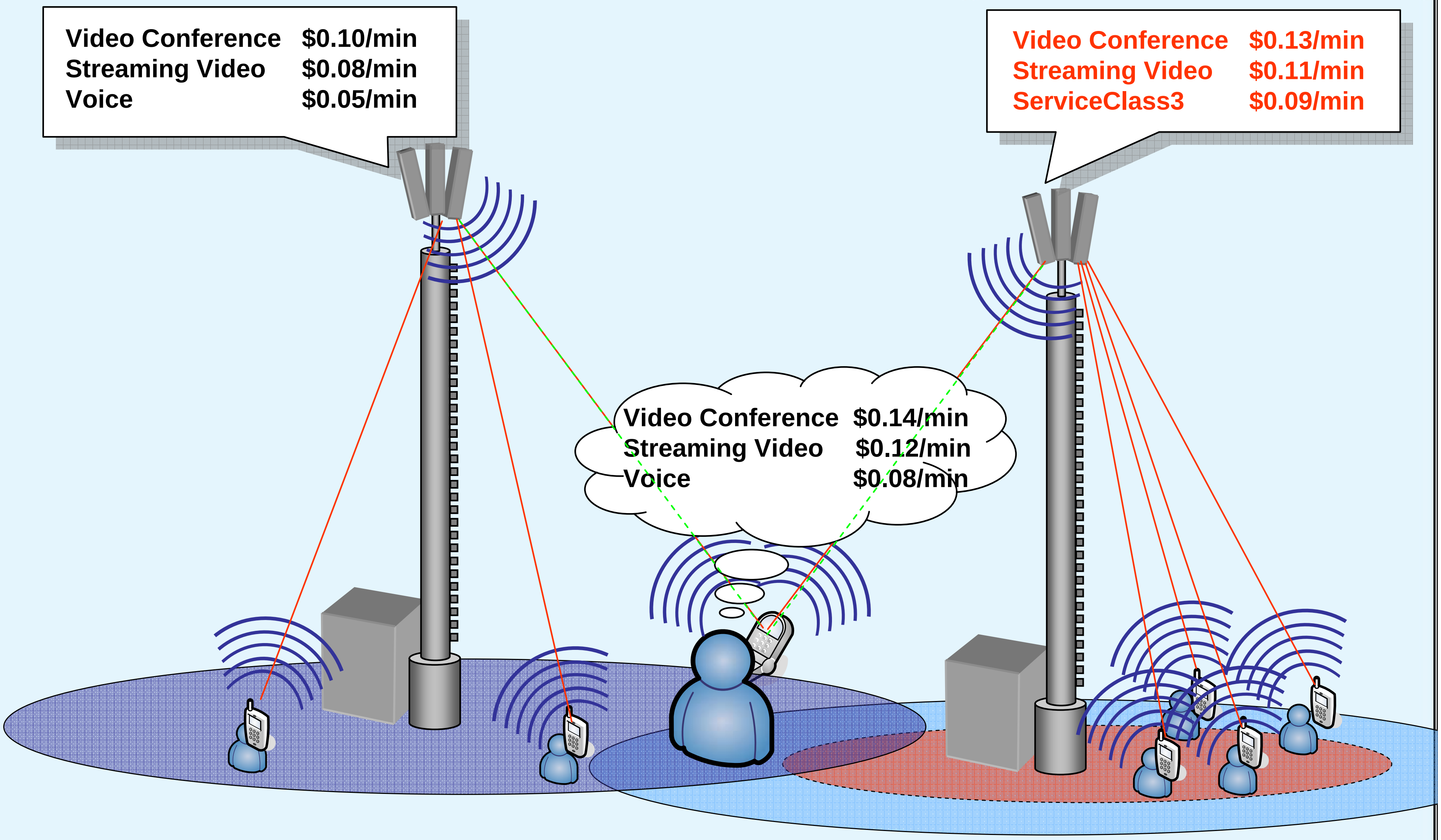


Logical and physical layers allowing a layered view of dynamic pricing

	Description	Motivation
Subscription	Design of price plans to let customers switch from competing providers in long-term	Provider revenue
Admission	Prices dynamically formed at the time of request depending on situation in network	Provider revenue
Flow/session	Pricing of bandwidth shares to manage congestion and maximise allocation efficiency	Economic efficiency
Packet	Price attached with each packet to set priority in times of congestion (e.g., smart market)	Technical and economic efficiency
Physical unit	Use of pricing to allocate radio resources such as power or transmission slots	Technical efficiency (Engineering)

Background and related literature: An integrated-service model by Wang et al. (97)

- Maximise revenue for N service classes and one best-effort class
- Price p_i as time-based charge for using the service class i
- Price p_b as a price per packet of best effort class b
- Optimal control model

$$max_{p_i,p_b \leq 0} \int_0^T \{ \sum_{i=1,N} [1-\beta_i] p_i(t) \frac{\lambda_i(p_i(t),t)}{r_i} + p_b \lambda_b(p_b(t),t)\} dt$$

- Where $\lambda_i(p_i,t)$ is the arrival rate in class i, $\lambda_b(p_b,t)$ is the packet arrival rate, 1/r is the service duration, and β_i is the target blocking rate

- Optimal prices

$$p_i^*(t) = \frac{\epsilon_i}{\epsilon_i - 1} h_i(t),\ \ p_b^*(t) = \frac{\epsilon_b}{\epsilon_b - 1} l_2(t)$$

- Where ε is the price elasticity of demand, $l_2(t)$ is the Lagrange multiplier, and $h_i(t)$ is the Hamiltonian of the state variables

Summary of assumptions

- Two providers, each operating a single cell
- Customers are location stationary during the time of service usage
- All customers can access both networks

- Only consider the forward link of a W-CDMA based cell
- Constant bit rate services with predefined bandwidth and QoS, which is defined by a maximum average BER

- Service activation rate $\lambda_z(x(t))$ for a circular area with radius z
- Mean service duration 1/r independent of price
- $1 - \alpha(t_1, t_2)$ denotes the cell overlap, which is determined by the positions of the base stations t_1 and t_2

In the game of complete information the entire cell setup is common knowledge

- Game played between two providers
- Cell setup (position of base stations t_i, capacity C_{max}, resource constraints) is common knowledge

$$R_i = x_i \frac{\lambda_z(x_i)}{r}\left(\alpha(t_i, t_j) + (1-\alpha(t_i,t_j))\beta(x_i,x_j)\right)$$

- Where $\alpha(t_i, t_j)$ is the percentage of the cell not overlapped

$$\beta_i(x_i,x_j) = \begin{cases} 1 & \text{if } x_i < x_j \\ \frac{1}{2} & \text{if } x_i = x_j \\ 0 & \text{if } x_i > x_j \end{cases}$$

- Search for reaction function and Nash equilibria

- Depending on the value for β, the solutions are given with

$$x_i = \begin{cases} x_A = \frac{e}{2f},\ x_B = \frac{e}{f} - \frac{rC_{max}}{cf} & \text{for } \beta = 0 \\ x_A = \frac{e}{2f},\ x_C = \frac{e}{f} - \frac{2rC_{max}}{cf(1+\alpha)} & \text{for } \beta = \frac{1}{2} \\ x_A = \frac{e}{2f},\ x_D = \frac{e}{f} - \frac{rC_{max}}{cf\alpha} & \text{for } \beta = 1 \end{cases}$$

- Four cases:

$$x_B \geq x_C \geq x_A$$

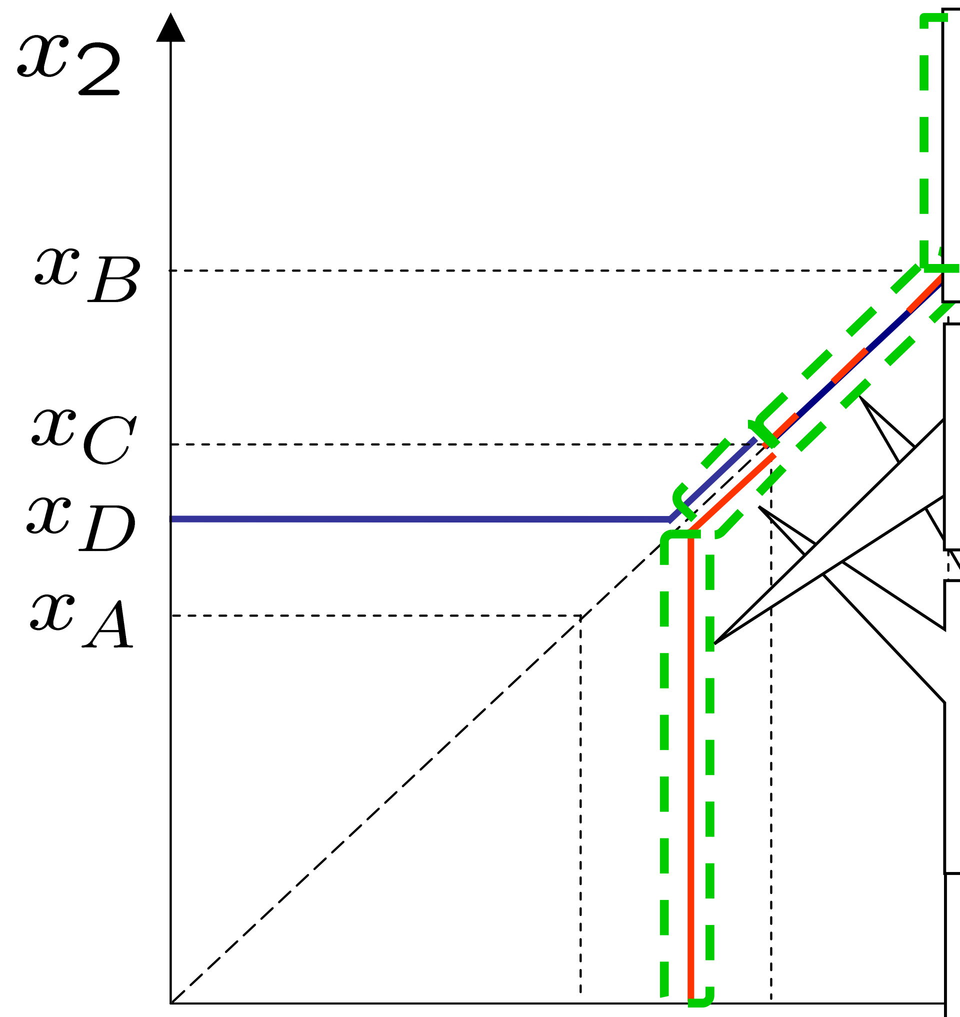
$$x_B \geq x_A \geq x_C$$

$$x_A \geq x_B \geq x_E$$

$$x_B \leq x_E \vee x_B < 0$$

$$x_E = \frac{e}{2f} - \frac{\sqrt{e^2 f^2 (1-\alpha^2)}}{2f^2(1+\alpha)}$$

Finding equilibria



Player 1 sets his price equal to x_D and only serves the customers outside the overlapping area

Player 1 sets his price equal to the revenue-maximising price x_B

Player 1 needs to set a price slightly higher than the competitor to not violate the resource constraint

"share" customers in the overlapping area. No player wants to unilaterally deviate

A Bayesian game

- Location of its base station t_i is player i's private information
- Distribution of t_i over T_i is common knowledge
- Provider 1 must solve

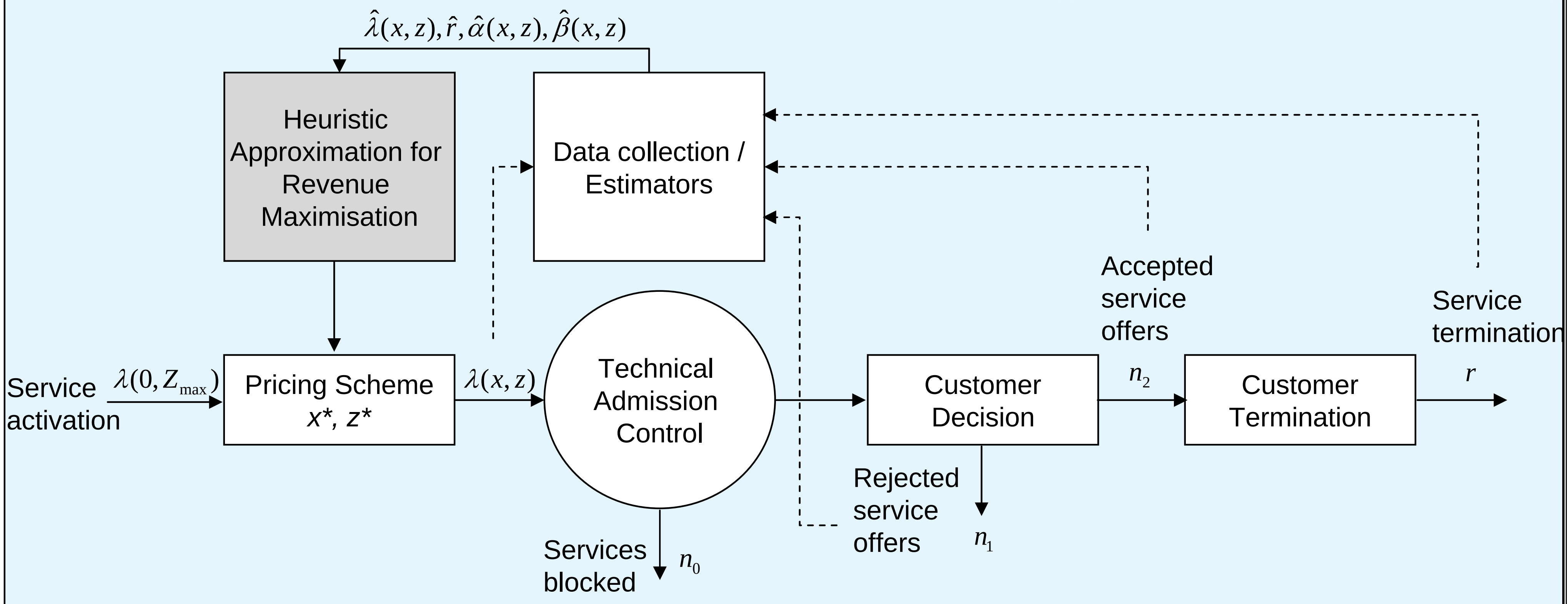
$$\max_{x_i} \mathbb{E}_i[R_i(x_i)] = x_i \frac{\lambda(x_i)}{r} \int_{t_2} \left[\alpha + (1-\alpha)\,\mathrm{Prob}\{x_i < x_j\}\right] dt_2$$

- Is there an equilibrium pricing strategy, which maximises a players payoff for all his types t_i in T_i?

Approaches

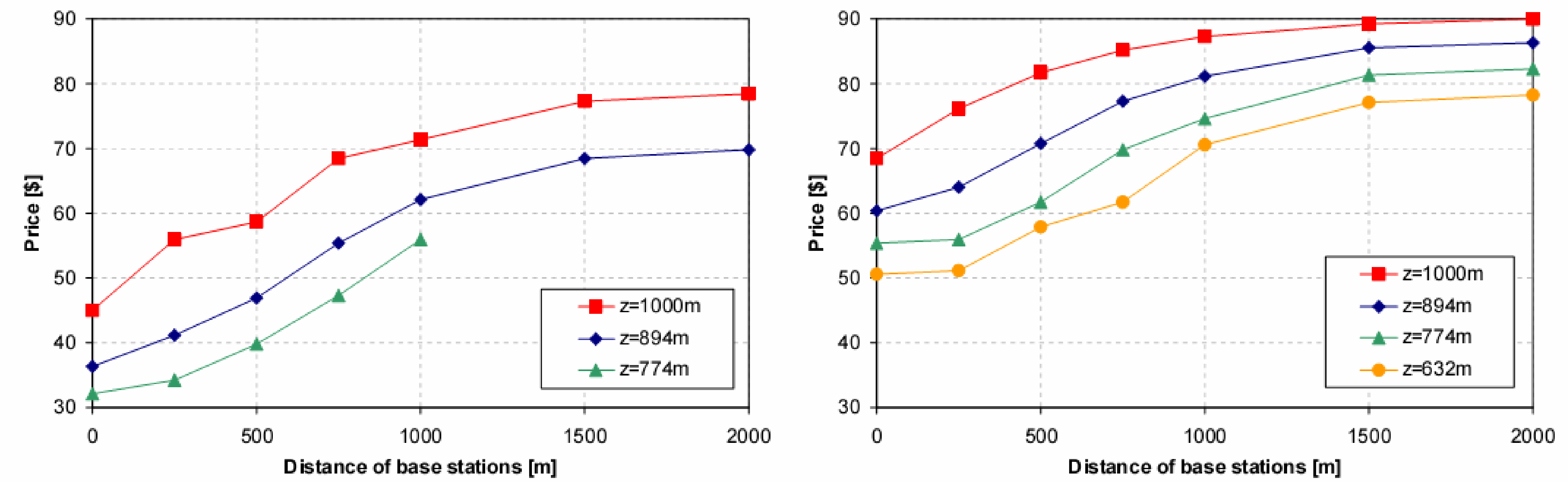
- Does there exist an equilibrium in linear pricing strategies in the form $x_i(t_i)=b_i-c_it_i$?
- Does there exist an equilibrium in hyperbolic pricing strategies in the form $x_i(t_i)= b_i-c_i/(1+t_i)$?
- Does there exist an equilibrium in symmetric pricing strategies in the form $x_i(t_i)=s(.)$?
- None of them could be found

Alternatively we build a simulation model



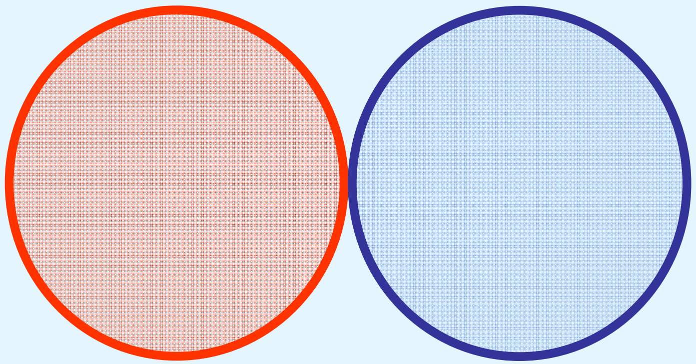
- Use of a heuristic approximation to find near-optimal solution for the constrained maximisation problem
- Form estimators about demand, customer access options, competitive prices
- Use of a sliding-window technique to produce estimators

Price – cell-radius combinations when varying the position of one base station



(a) Price - cell-radius selection against the distance of the base stations for a user density of 100 users/km^2.

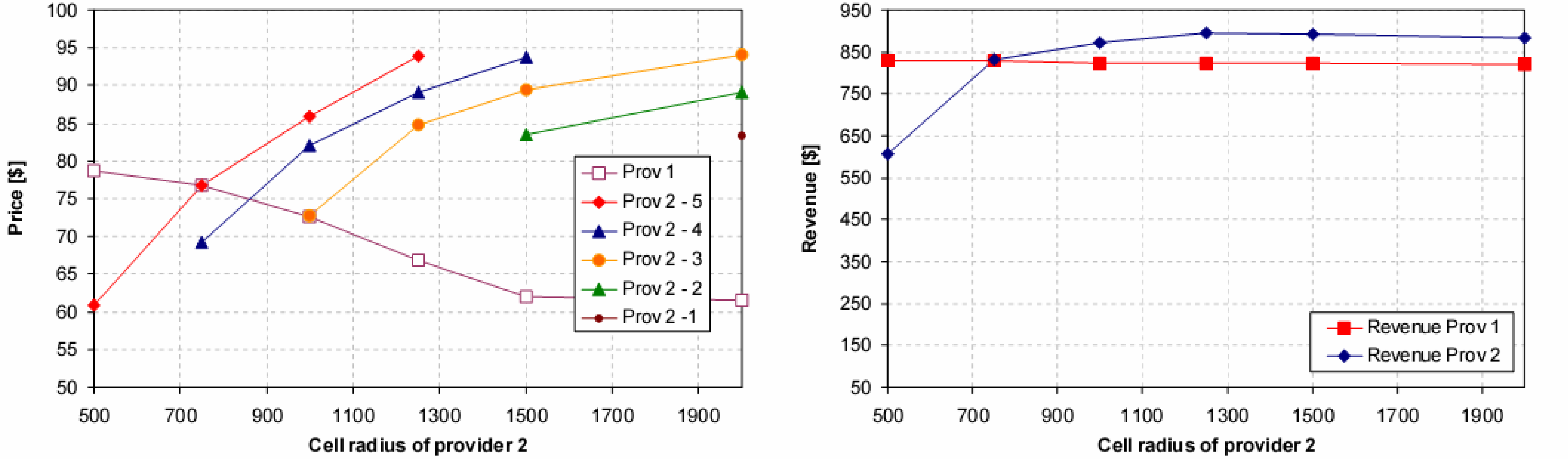
(b) Price - cell-radius selection against the distance of the base stations for a user density of 200 users/km^2.



Simulation Setup:

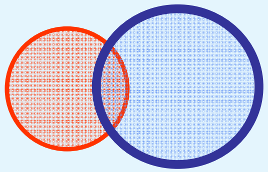
- Maximum cell radius Z_{max}=1,000m
- Constant-Bit-Rate 144kbps / 1.00E-6
- Chip Rate 3.84Mcps
- Transmission Power 30Watts

Price – cell-radius combinations when introducing single-access customers in one network



(a) Price/cell-radius combinations for provider 1 and 2 against the cell size of provider 2.

(b) Average revenue for provider 1 and 2 against the cell size of provider 2.



Simulation Setup:

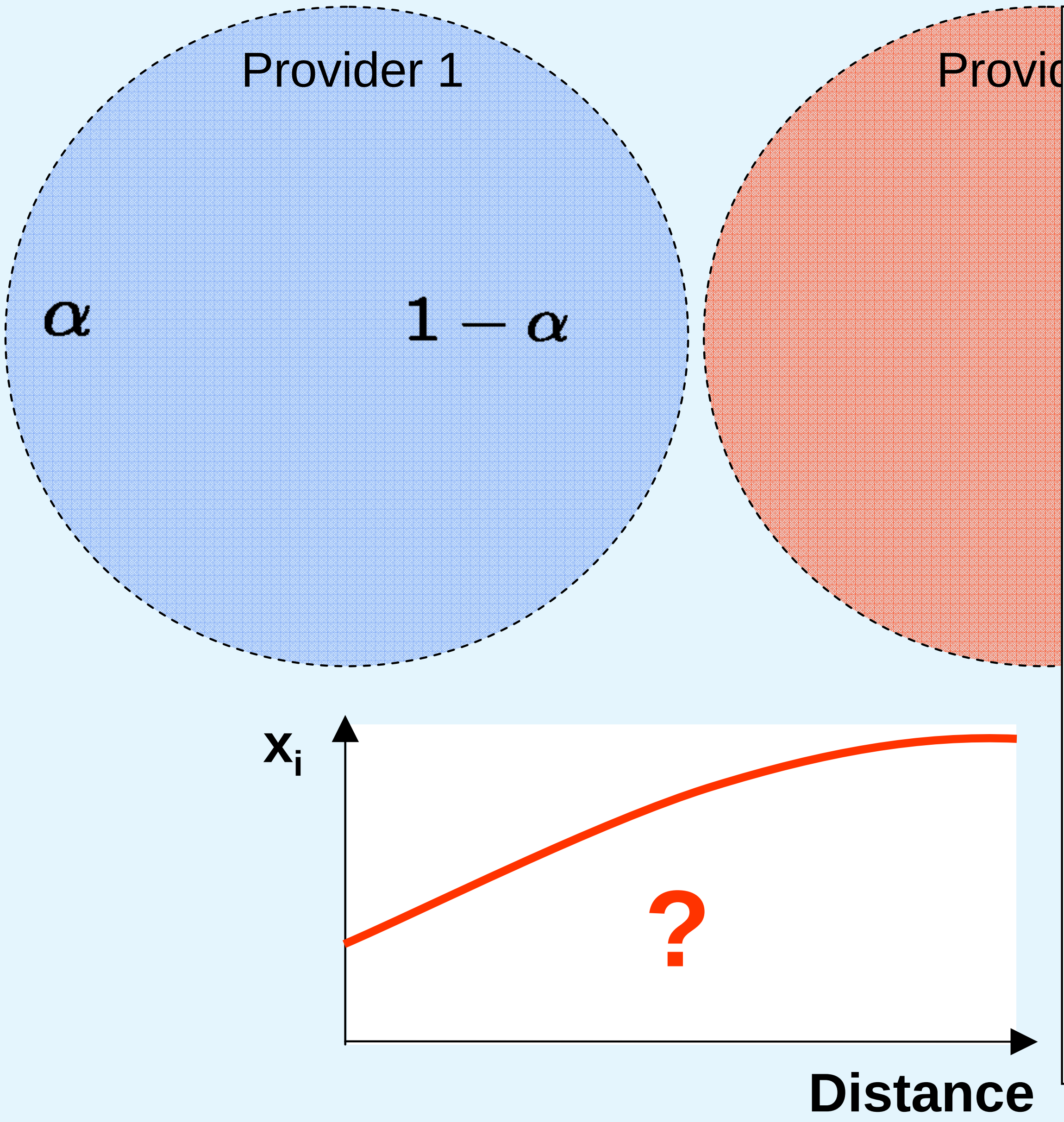
- Maximum cell radius Z_{max}=750m (Prov 1)
- Changing cell radius of Prov 2 from 500m to 2,000m
- Distance between base stations d=1,000m
- Identical setup to last experiment

Conclusions

- Game of complete information, with capacity constraints, provides a Bertrand-like solution to the problem
- Solution to Bayesian game provides only partial insights on the “best-response” function
- Simulation platform developed as a means to gain some more understanding of the strategic aspects of the problem
- Simulation results provide some indication about the direction in which equlibria could be found.
- In spite of the simplified assumptions, we believe this approach can be further pursued to build upon the “feedback” between simulation and analytical models of competition.

BACKUP

The provider problem under competition can be modeled as a game



Revenue determined by

- Customers served monopolistically α
- Customers under competition $1-\alpha$
- Price relative to competitive price

$$\beta_i(x_i, x_j) = \text{Prob}\{x_i < x_j\}$$

- Expected revenue becomes

$$R = x_i \frac{\lambda(x_i)}{r}(\alpha + (1-\alpha)\beta)$$

- How to find the equilibrium price function $x_i(t_i)$?

The heuristic approximation model for revenue maximisation

Revenue for all (x,d) combinations

	$0	$30	$50	$70	$90
100m	$0	$200	$140	$98	$69
300m	$0	$600	$420	$294	$206
500m	$0	$1,230	$861	$603	$422
700m	$0	$2,190	$1,533	$1,073	$751
900m	$0	$4,530	$3,171	$2,220	$1,554

Rate Capacity Constraint

	$0	$30	$50	$70	$90
100m					
300m					
500m					
700m					
900m					

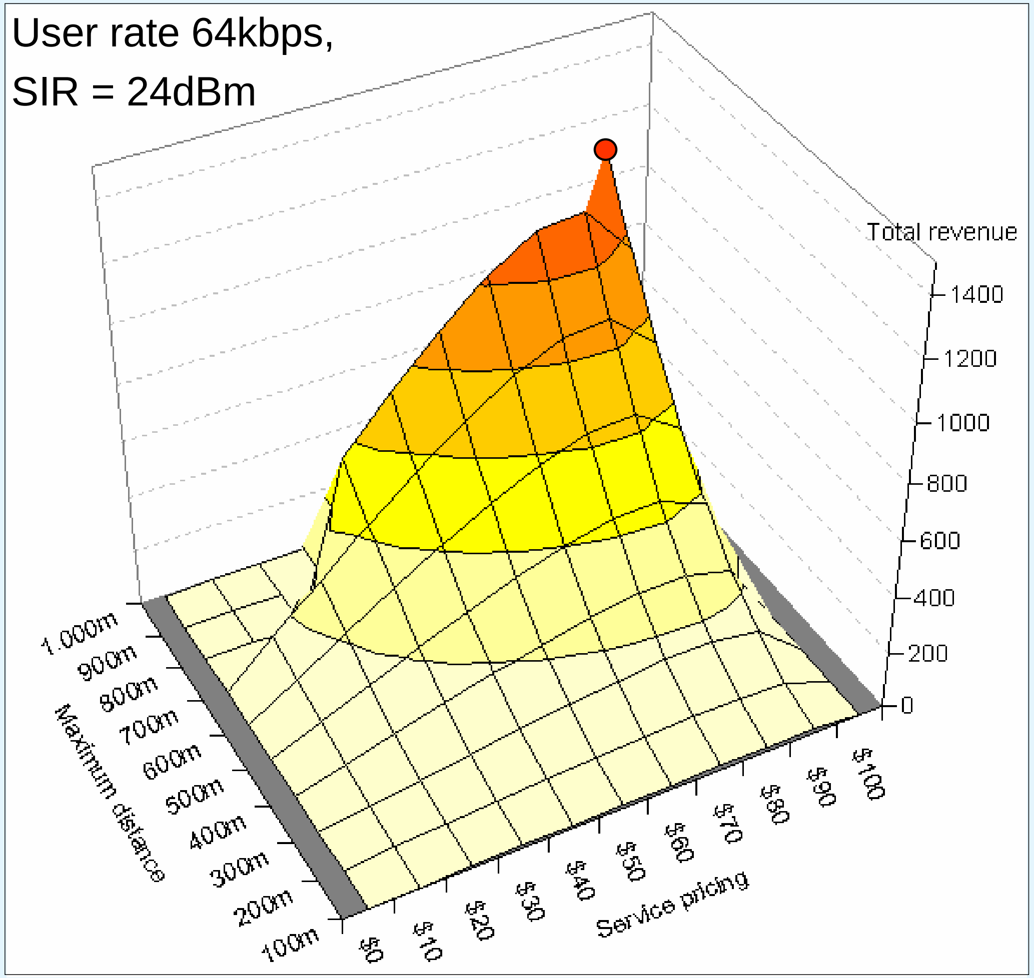
Power Capacity Constraint

	$0	$30	$50	$70	$90
100m					
300m					
500m					
700m					
900m					

Feasible revenue

	$0	$30	$50	$70	$90
100m	0	200	140	98	69
300m	0	600	420	294	206
500m	0	1230	861	603	422
700m	0	0	1533	1073	751
900m	0	0	0	0	0

Graphical representation



We use simulation to learn about the outcomes of the game in different settings

- Implement heuristic approximation to find near optimal solution with price and cell radius
- Let providers learn about market situation
  - Estimators of customer arrival
  - Signals about competitive situation
  - Measurement of blocking ratios and rejection ratios
- Model a stochastic environment
- Run multiple replications and use statistics to draw conclusions

- Use of agent-based simulation
- Each entity (network cell, customer) represented by agent
- Each running individual routines
- Very flexible, extendible