# Pricing in telecommunication networks: some issues and models

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# Outline

# Internet alone: why changing the pricing scheme?

- Increase of Internet traffic due to
  - increasing number of subscribers
  - more and more demanding applications.
- Congestion is a consequence, with erratic QoS.
- Increasing capacity difficult if not impossible in access networks (last mile problem).
- Also, *flat rate* pricing unfair and does not allow service differentiation. Subject of debate...
- Several works on this issue (service differentiation, auctions for bandwidth...)
- Convergence (services and technologies) requires new pricing offers: bundles to attract customers to services they would not consider otherwise.
- New aspect: network neutrality (and regulation in general): introduce limitations. Modeling the impact?
- To implement BoD, a pricing/charging scheme to be attached.

# General modelling tools

- To represent network performance: queueing analysis or signal processing.
- Network made of non-cooperative units (agents) trying to maximize their own "benefit".
- But your action has an influence on others': so-called *positive or negative externalities*.
- To represent the interactions of selfish nodes in communication networks: game theory.
  - Each player $i$ (user or provider) represented by its utility function $U_i(x)$ representing quantitatively its level of satisfaction (in monetary units for instance) when actions profile is $x = (x_i)_i$.
  - Basic notion: Nash equilibrium: an action profile $x^* = (x_i^*)_i$ such that no player can unilaterally increase his utility:

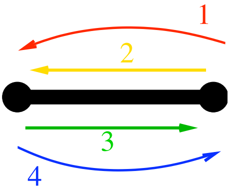
$$U_i(x^*) \geq U_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots) \qquad \forall x_i.$$

# Proposed pricing schemes

- Pricing for guaranteed services through reservation and admission control (drawback: scalability).
- *Paris Metro Pricing*: separate the network into logical subnetworks with different acces charges (advantage: simple. Drawback: does not work in a competion market).
- *Cumulus pricing scheme*: +/- points awarded depending on the respect of the predefined contract. Penalities and renegociations when thresholds reached (advantage: easy to implement).
- Priority pricing: classes of traffic with different priority levels and access prices (advantage: easy to implement)
  - schedulling priority
  - rejection or dropping priority.
- Auctionning, for priority at the packet level, or for bandwidth at the flow level.
- Pricing based on transfer rates and shadow prices.

# Example: auctionning for bandwidth

**The problem of resource allocation**




- Allocate bandwidth among users on a link with a capacity constraint $Q$
- More general results also obtained
- Allocation and pricing mechanism: determines the allocation $a_i$ for each player $i$, and the price $c_i$ he is charged.

# Multi-bid auctions

Maillé & T., Infocom 04, IEEE/ACM ToN 06

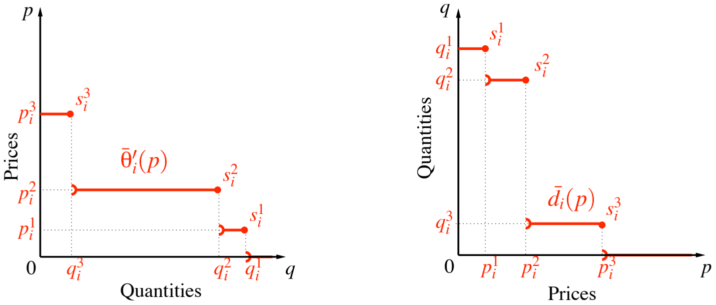
- When entering the game, each player $i$ submits $M_i$ two-dimensional bids of the form $s_i^{m_i} = (q_i^{m_i}, p_i^{m_i})$ where

$$\begin{cases} q_i^j &= \text{asked quantity of resource} \\ p_i^j &= \text{corresponding proposed unit price} \end{cases}$$

- Allocations $a_i$ and charges $c_i$ computed based on $s$.

# User behaviour

- Set $\mathcal{I}$ of users (players)
  - Users' preferences: determined by their **utility function** $u_i(s) = \theta_i(a_i(s)) - c_i(s)$
  - $\theta_i$ =player $i$'s **valuation function**, assumed non-decreasing and concave
  - User $i$'s goal: maximizing his utility $\theta_i(a_i) - c_i$.
- The auctioneer uses player $i$'s multi-bid $s_i$ to compute:
  - the pseudo-marginal valuation function $\bar{\theta}'_i$
  - the pseudo-demand function $\bar{d}_i$

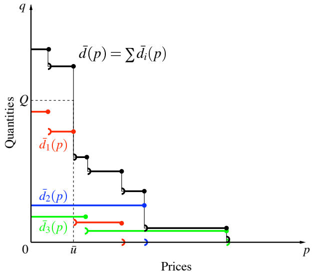



Pseudo-marginal valuation and pseudo-demand functions associated with the multi-bid $s_i$

$$\bar{\theta}_i'(q) = \max_{1\leq m\leq M_i}\{p_i^m : q_i^m \geq q\} \text{ if } q_i^1 \geq q, \qquad 0 \text{ otherwise.}$$

$$\bar{d}_i(p) = \max_{1\leq m\leq M_i}\{q_i^m : p_i^m \geq p\} \text{ if } p_i^{M_i} < p, \qquad 0 \text{ otherwise.}$$

# Allocation rule




$\bar{u}$: pseudo market clearing price (highest unit price at which demand exceeds capacity).

- Multi-bid allocation: $a_i(s) = \bar{d}_i(\bar{u}^+) + \frac{\bar{d}_i(\bar{u}) - \bar{d}_i(\bar{u}^+)}{\bar{d}(\bar{u}) - \bar{d}(\bar{u}^+)}(Q - \bar{d}(\bar{u}^+))$
- Interpretation:
  - If $i$ was allocated $\bar{d}_i(\bar{u})$, it may happen that too much is allocted; if $\bar{d}_i(\bar{u}^+)$, maybe not everything!
  - We therefore choose to allocate $\bar{d}_i(\bar{u}^+)$ and share the remaining part $(Q - \bar{d}(\bar{u}^+))$ proportionally to the demand level.

# Pricing rule: Vickrey-Clarke-Groves (VCG) mechanism

- Pricing principle : each user pays for the declared "social opportunity cost" he imposes on others
- In other words: you pay the difference in terms of the sum of (pseudo-) valuation of players your presence creates.
- In case of a single (atomic) item: as the winner (highest bidder), you would pay the second-highest bid.
- If $s$ denotes the bid profile,

$$c_i(s) = \sum_{j \in \mathcal{I} \cup \{0\}, j \neq i} \int_{a_j(s)}^{a_j(s_{-i})} \bar{\theta}'_j$$

# Properties of the scheme

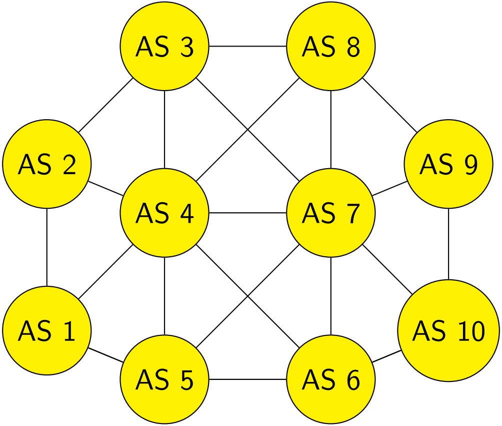
- *a) Incentive compatibility*: A player cannot do much better than simply revealing his valuation.
- *b) Individual rationality*: $U_i \geq 0$, whatever the other players bid.
- *c) Efficiency*: When players submit truthful bids, the allocation maximizes social welfare.

Other advantages of the scheme:

- No information required about others when submitting bids
- Multi-bids submitted only once and for all.
- "Robust" to players entering and leaving the game.

If $M_i = 1$, only one bid, possibility of convergence but requires to know the whole set of bids, and several rounds Semret et al.

# Interdomain: key problem for BoD




- Network made of Autonomous Systems (ASes) acting selfishly.

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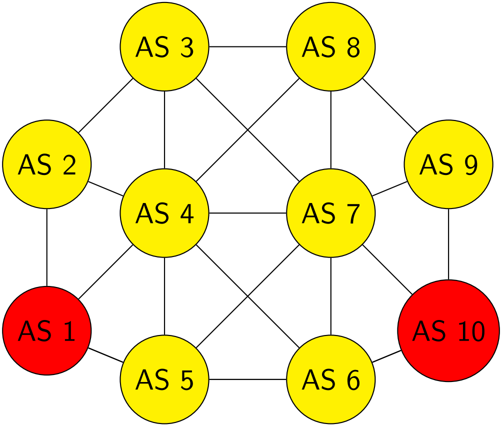


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- A node (an AS) needs to send traffic from its own customers to other ASes.
- Introduce incentives for intermediate nodes to forward traffic , via pricing.

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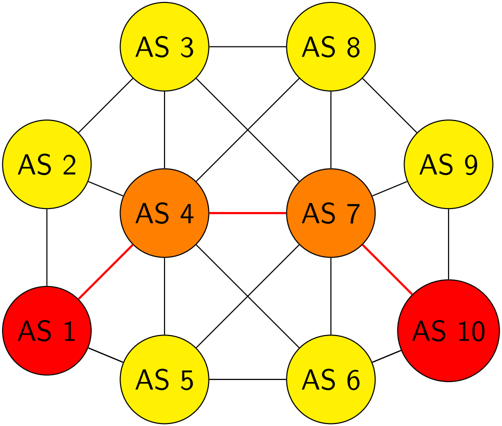


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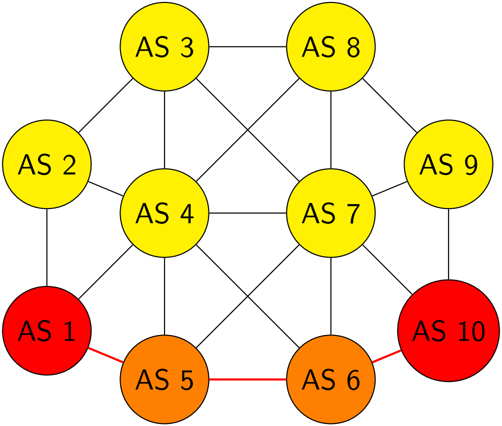


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# Interdomain issues

- similar problems in
  - ad-hoc networks: individual nodes should be rewarded for forwarding traffic (especially due to power use);
  - P2P systems: free riding can be avoided through pricing.
- How to implement it?
  - The AS can contacts all potential ASes on a path to learn their costs, and then make its decisions.
  - More likely: he contacts only its neighbors, which ask the cost to their own neighbors with a BGP-based algorithm.
    On the way back, declared costs are added.
- Two different mathematical problems
  - Finite capacity at each AS: it becomes similar to a knapsack problem.
  - Capacity assumed infinite if networks overprovisionned thanks to optic fiber (last mile problem, i.e., connection to users, not considered here).

# Relevant (desirable) properties

- Individual rationality: ensures that participating to the game will give non-negative utility.
- Incentive compatibility: ASes' best interest is to declare their real costs.
- Efficiency: mechanism results in a maximized sum of utilities.
- Budget Balance: sum of money exchanged is null.
- Decentralized: decentralized implementation of the mechanism.
- Collusion robustness: no incentive to collusion among ASes.

Is there a pricing mechanism:

- verifying the whole set or a given set of properties?
- Or/and verifying *almost* all of them?

# Interdomain pricing when no resource constraints

Feigenbaum et al. 2002

- Inter-domain routing handled by a simple modification of BGP.
- Amount of traffic $T_{ij}$ from AS $i$ to AS $j$, with per-unit cost $c_k$ for forwarding for AS $k$.
- Valuation of intermediate domain $k$ for a given allocation (a routing decision) is

$$\theta_k(\text{routing}) = -c_k \sum_{\{(i,j) \text{ routed trough } k\}} T_{ij}.$$

- Maximizing sum of utilities is equivalent to minimizing the total routing cost

$$\sum_{i,j} T_{ij} \sum_{k \in path(i,j)} c_k,$$

where
  - each AS declares its transit cost $c_k$
  - the least (declared) cost route $path(i,j)$ is computed for each origin-destination pair $(i,j)$.

# VCG auctions and drawback in interdomain context

- Payment rule to intermediate node $k$ (opportunity cost-based):

$$p_k = c_k + \left( \sum_{\ell \text{ on } path^{-k}(i,j)} c_\ell - \sum_{\ell \text{ on } path(i,j)} c_\ell \right)$$

with $path^{-k}(i,j)$ the selected path when $k$ declares an infinite cost.

- Subsequent properties
  - Efficiency
  - Incentive compatibility
  - Individual rationality
- Only pricing mechanism to provide the three properties at the same time.

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- Subsequent properties
  - Efficiency
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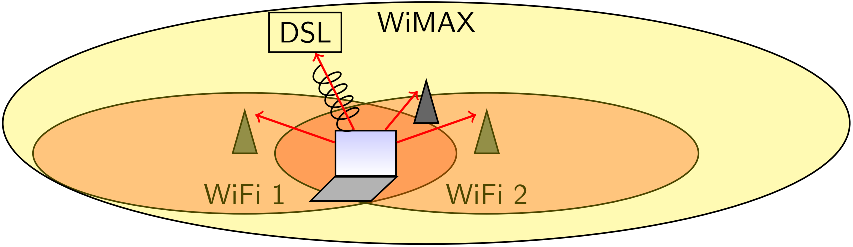
- Only pricing mechanism to provide the three properties at the same time.
- But who should pay the subsidies? Sender's willingness to pay not taken into account. That should be!
  The VCG payment from sender is the sum of declared costs if traffic is effectively sent: always below the sum of subsidies.
  Very unlikely to apply in practice: no central authority to permanently inject money.

# Impossibility result and what is the good choice?

- General result: no mechanism can actually verify efficiency, incentive compatibility, individual rationality and budget balance.
- Current question: what set of properties to verify? Which mechanism to apply?
  - The "almost" property could be amore flexible choice.
  - Strict requirement: budget balance. Decentralization too if dealing with large topologies.

# Specific model of competition among providers



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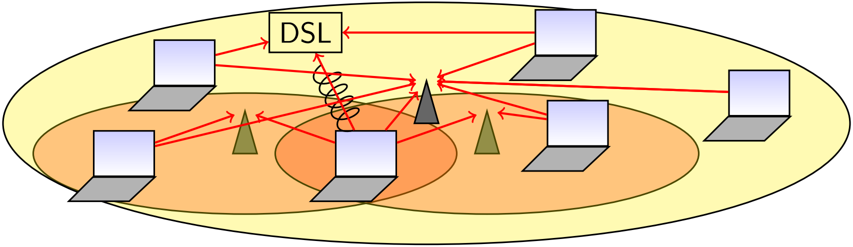
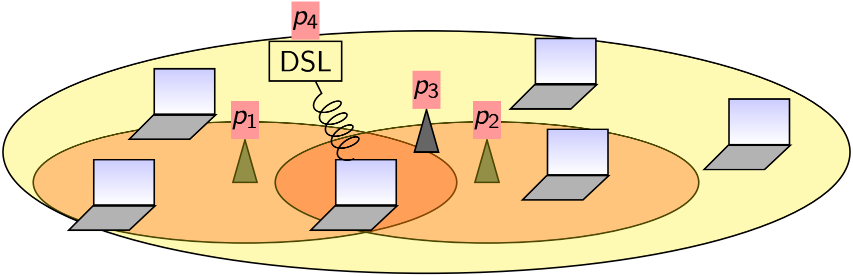



- Interactions among non-cooperative consumers: *game*
- Congested networks provide poorer quality (packet losses)

# Specific model of competition among providers

But **providers** play first!



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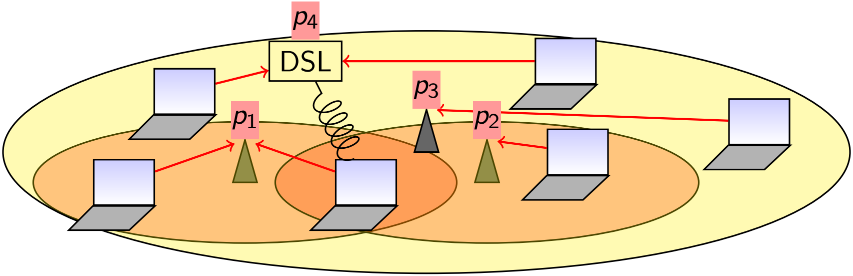
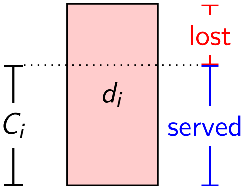



This work: study of the two-level noncooperative game.

1. *Higher level*: **providers** set prices to maximize revenue
2. *Lower level*: **consumers** choose their provider

# Communication model: packet losses

- Time is slotted
- Each provider $i$ has finite capacity $C_i$
- If total demand $d_i$ at provider $i$ exceeds $C_i$: exceeding packets are *randomly* lost




$$\mathbb{P}(\text{successful transmission}) = \min\left(1, \frac{C_i}{d_i}\right)$$

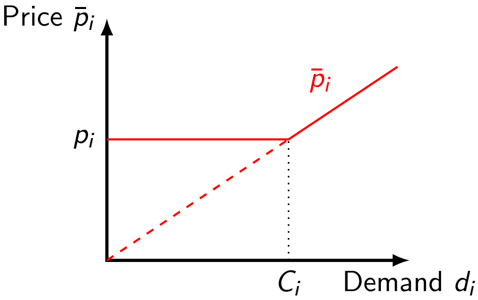
$$\Rightarrow \text{Expected number of transmissions} = \frac{1}{\mathbb{P}(\text{success})} = \max\left(1, \frac{d_i}{C_i}\right)$$

# Only "regulation": pay for what you send

The price $p_i$ at each provider $i$ is per packet *sent* Marbach'02

$\Rightarrow$ If several transmissions are needed, the user pays several times

$$\bar{p}_i := \textit{perceived} \text{ price at } i = \mathbb{E}[\text{price per packet}] = p_i \max\left(1, \frac{d_i}{C_i}\right)$$



# Model for user choices: Wardrop equilibrium

- Users choose the provider(s) $i$ with lowest $\bar{p}_i = p_i \max\left(1, \frac{d_i}{C_i}\right)$

$\Rightarrow$ For a given coverage zone $Z$, all providers with customers from that zone end up with the same perceived price $\bar{p}_i = \bar{p}_z$ Wardrop'52

- The total amount of data that users want to successfully transmit in a zone $z$ depends on that price:

$$\sum_i d_{i,z} \min(1, C_i/d_i) = \alpha_z D(\bar{p}_z),$$

$$i.e. \quad \bar{p}_z = \underbrace{v}_{\text{marg. val. function}} \left( \frac{\sum_i d_{i,z} \min(1, C_i/d_i)}{\alpha_z} \right)$$

with $D$ the total demand function, $\alpha_z$ the population proportion in zone $z$, and $d_{i,z}$ the demand in zone $z$ for provider $i$.

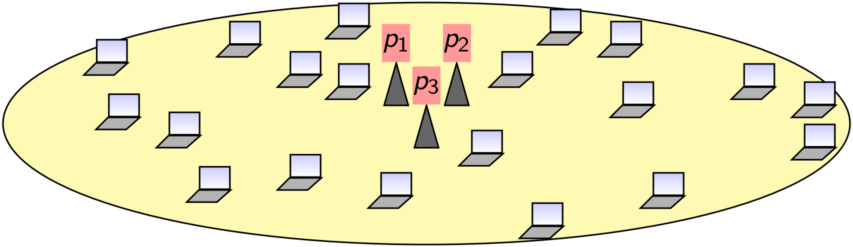
# Higher level: price competition game

- Providers set their price $p_i$ *anticipating users reaction*
  $\Rightarrow$ Providers are Stackelberg leaders
- We can assume management costs of the form $\underbrace{\ell_i(d_i)}_{\text{nondecreasing, convex}}$

  Provider $i$'s objective: $R_i := p_i d_i - \ell_i(d_i)$.

# Competition model

- Simplified topology: common coverage area
- $N$ competing providers declaring price and capacity ($\mathcal{I} := \{1, \ldots, N\}$)



# User equilibrium

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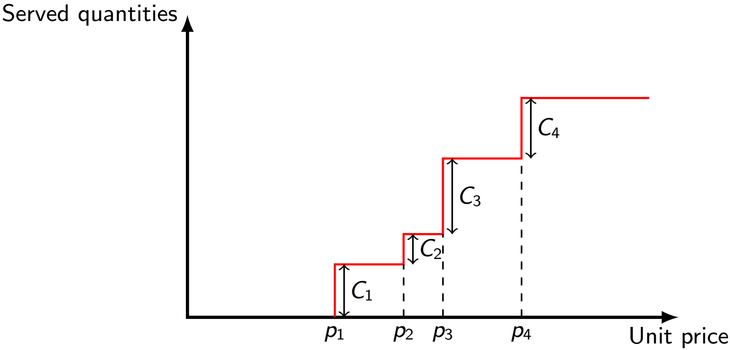

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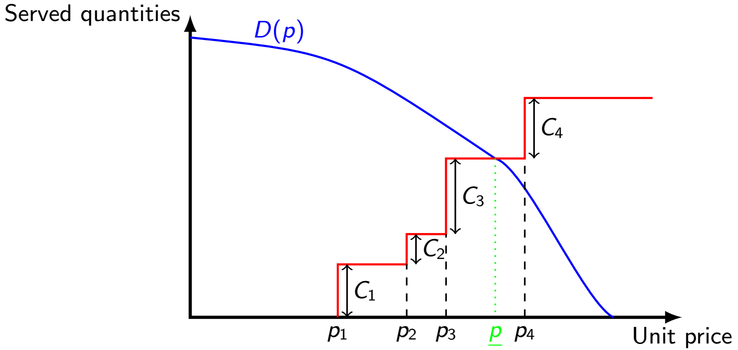
- The total demand level depends on that price:

$$\bar{p} = \underbrace{v}_{\text{marg. val. function}}\left(\sum \min(C_i, d_i)\right)$$



# Price competition, main result

## Proposition

*Under sufficient condition A, there exists a* **unique Nash equilibrium** *on price war among providers, given by*

$$\forall i \in \mathcal{I}, \quad \begin{cases} p_i &= v\left(\sum_{j\in\mathcal{I}} C_j\right) \\ d_i &= C_i. \end{cases}$$

- **Sufficient condition A:** each $\ell_i$ is Lipschitz with constant $\kappa_i$, and $\forall y \geq p^* := v\left(\sum_{j\in\mathcal{I}} C_j\right)$, the demand function $D$ is sufficiently elastic:
$$\frac{-yD'(y)}{D(y)} \geq \frac{1}{1-\kappa/y}, \tag{1}$$
where $\kappa := \max_{i\in\mathcal{I}} \kappa_i$.
- Without cost functions, it just means a demand elasticity larger than -1.

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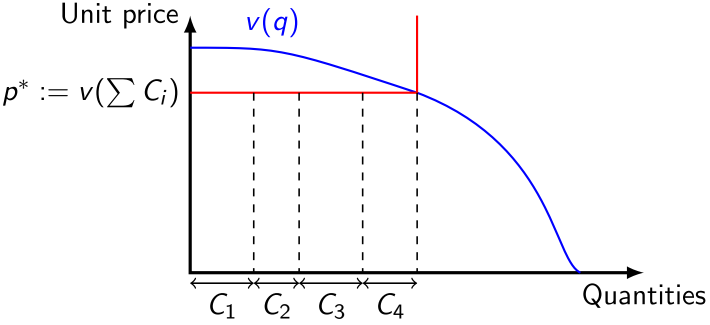


# Social Welfare considerations

- A performance measure of the outcome $(d_1, ..., d_I)$ of the game $=$ overall value of the system

$$\text{Social Welfare} := \int_{u=0}^{\sum_{i\in\mathcal{I}} d_i} \frac{\sum_{i\in\mathcal{I}} \min(d_i, C_i)}{\sum_{i\in\mathcal{I}} d_i} v(u)du - \sum_i \ell_i(d_i).$$

- **Remark:** the Social Welfare maximization problem leads to the same outcome $d_i = C_i \quad \forall i$ as the price war.
- **Consequence:** The Nash equilibrium corresponds to the socially optimal situation: the Price of Anarchy is 1!.

# Game on declared capacities: a third level

We now consider a 3-stage game:

1. Providers $i \in \mathcal{I}$ declare their capacity $C_i$
2. Providers fix their selling price $p_i$
3. Users select their providers

Opposite effects of lowering one's capacity:

- the unit selling price at equilibrium increases and the managing cost decreases because the quantity sold decreases
- whereas on the other hand less quantity sold means less revenue.

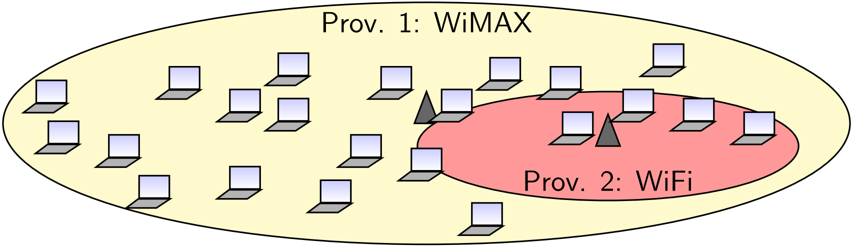
**Proposition**

*Under the same conditions about* **demand elasticity**, *no provider can increase its revenue by artificially lowering its capacity.*

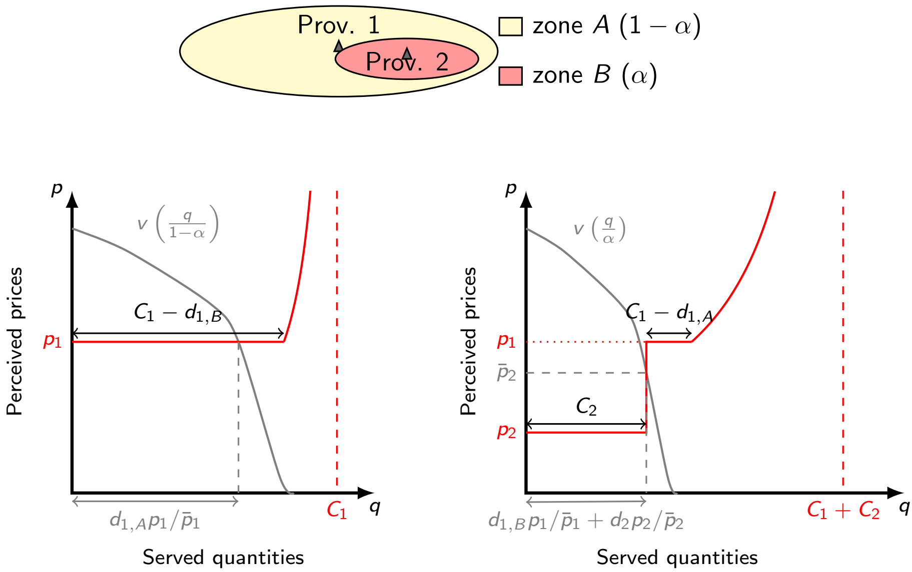
# Competition model

Assumptions

- Two competing providers declaring price and capacity
- One coverage area included in the other



# User equilibrium: illustration



# User equilibrium: mathematical formulation

At user equilibrium, according to Wardrop principle

$$\begin{aligned}
\bar{p}_1 &= p_1 \max\left(1, \frac{d_{1,A}+d_{1,B}}{C_1}\right) \\
\bar{p}_2 &= p_2 \max\left(1, \frac{d_2}{C_2}\right)
\end{aligned}$$

$$\begin{aligned}
d_{1,A} \min\left(1, \frac{C_1}{d_{1,A}+d_{1,B}}\right) &= (1-\alpha)D(\bar{p}_1) \\
d_{1,B} \min\left(1, \frac{C_1}{d_{1,A}+d_{1,B}}\right) + d_2 \min(1, C_2/d_2) &= \alpha D(\min(\bar{p}_1, \bar{p}_2))
\end{aligned}$$

$$\begin{aligned}
\bar{p}_1 > \bar{p}_2 &\Rightarrow d_{1,B} = 0 \\
\bar{p}_1 < \bar{p}_2 &\Rightarrow d_2 = 0.
\end{aligned}$$

**User equilibrium: existence and uniqueness**

**Proposition**

*For all price profile, there exists at least a user (Wardrop) equilibrium. Moreover, the corresponding perceived prices of each provider are unique.*

*NB:* demand repartition among providers is not necessarily unique.

**Higher level: price competition game**

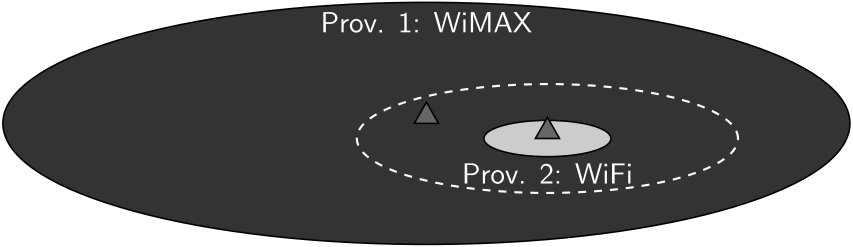
- Provider $i$'s objective: $R_i := p_i d_i - \ell_i(d_i)$.

## Proposition

*If* $-\frac{D'(p)p}{D(p)} > 1, \quad \forall p$ *(elastic demand), then there exists a unique Nash equilibrium* $(p_1^*, p_2^*)$ *in the price war between providers.*

- *If* $\alpha \leq \frac{C_2}{C_1+C_2}$, *then* $p_1^* = v\left(\frac{C_1}{1-\alpha}\right) \geq p_2^* = v\left(\frac{C_2}{\alpha}\right)$. *The common zone is left to provider* 2 *by provider* 1.
- *If* $\alpha > \frac{C_2}{C_1+C_2}$ *then* $p_1^* = p_2^* = p^* = v(C_1 + C_2)$. *The common zone is shared by the providers.*

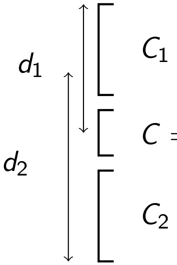
*(Darker=more expensive)*



# Partial spectrum sharing

Again one common coverage area and two providers, but **an amount $C$ of spectrum has to be shared among providers**

- Each provider $i$ still has some "private" band $C_i$
- If $d_i > C_i$, demand in excess $d_i - C_i$ is sent to the shared band.
- The shared spectrum is allocated in proportion with the providers' excess demand



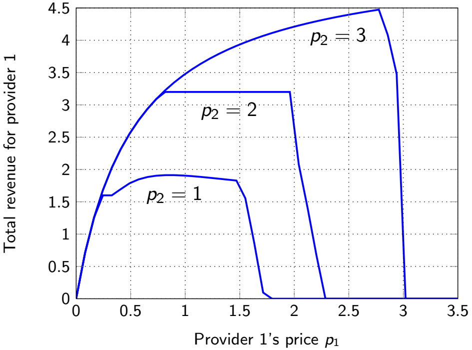

$$C = \begin{cases} C'_1 = \frac{[d_1 - C_1]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C \\ C'_2 = \frac{[d_2 - C_2]^+}{[d_1 - C_1]^+ + [d_2 - C_2]^+} C \end{cases}$$

## Proposition

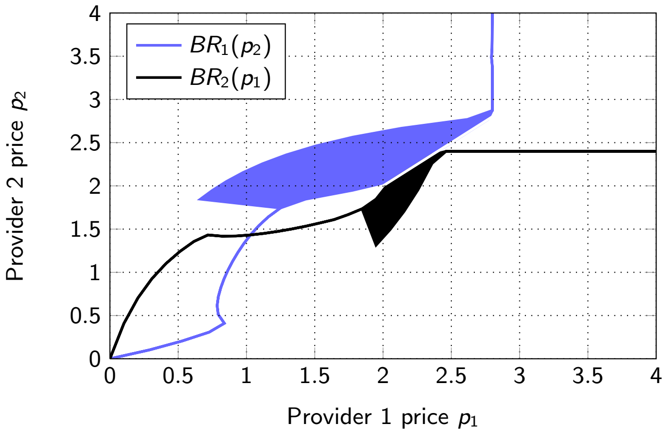
*Whatever the price profile $(p_1, p_2)$, there exists at least one Wardrop equilibrium. The corresponding perceived prices are unique.*

## Provider utilities:

$$R_i(p_1, p_2) := p_i d_i \text{ for } i \in \{1, 2\}.$$



# Provider best-reply curves




**Proposition**

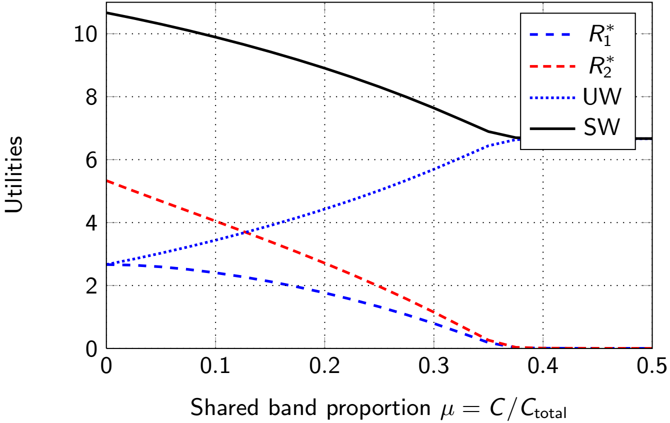
*There is no Nash equilibrium without losses.*

# Social Welfare considerations

The Social Welfare at Nash equilibrium is

$$\mathrm{SW} = \min\left(1, \frac{C_1 + C_2 + C}{D(\bar{p})}\right) \int_0^{D(\bar{p})} v, \qquad (2)$$

Influence of the fraction $\mu$ of total available band that is unlicensed?



# Conclusions

- Pricing an important issue in telecommunication networks, especially when dealing with BoD
- Many questions still open
  - Interdomain...
- New technological advances fostering pricing models
  - Ex: dynamic allocation of bandwidth in wireless networks
  - Ex: in security; pricing different security levels in a competitive environment.
- Network neutrality (political) issue and implication on BoD?
- New economical models
  - Ex: Mobile Network Operators (MNOs) leasing part of their network to Mobile Virtual Network Operators (MVNOs)? Interest for MNOs to do that? How to make it viable for MVNOs?
  - Ex: Mathematical models for preventing retention times of providers in competitive contexts.

# Few references, see also

http://www.irisa.fr/dionysos/pages_perso/tuffin/publications.html

## Multi-bid auctions:

- N. Semret. Market Mechanisms for Network Resource Sharing. PhD Thesis, Columbia University, 1998.
- P. Maillé, B. Tuffin. Multi-Bid Auctions for Bandwidth Allocation in Communication Networks. In *IEEE INFOCOM*, Hong-Kong, March 2004.
- P. Maillé and B. Tuffin. Pricing the Internet with Multi-Bid Auctions. *IEEE/ACM Transactions on Networking*, Vol. 14, Num. 5, pages 992-1104, 2006.

## Inter-domain issues:

- J. Feigenbaum, C. Papadimitriou, R. Sami, and S. Shenker. A BGP-based mechanism for lowest-cost routing. In *Proc. of the 21st ACM Symposium on Principles of Distributed Computing*, 2002.
- P. Maillé and B. Tuffin. Why VCG auctions can hardly be applied to the pricing of inter-domain and ad hoc networks. In *Proc. of the 3rd Conference on Next Generation Internet Networks (NGI 2007)*. Trondheim, Norway. May 21-23 2007.

## Competition:

- P. Marbach, Analysis of a static pricing scheme for priority services, *IEEE/ACM Transactions on Networking* 12 (2).
- J. Wardrop, Some theoretical aspects of road traffic research, *Proc. of the Institute of Civil Engineers*, 1952.
- P. Maillé and B. Tuffin. Competition Among Providers in Loss Networks. Submitted.
- P. Maillé and B. Tuffin. Price War in Heterogeneous Wireless Networks. *Computer Networks*. To appear.
- P. Maillé and B. Tuffin. Price War with Partial Spectrum Sharing for Competitive Wireless Service Providers.In *Proceedings of IEEE Globecom 2009*, Honolulu, USA, December 2009.