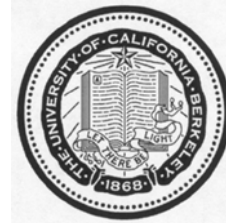

An Efficient Mechanism for Network Bandwidth Auction

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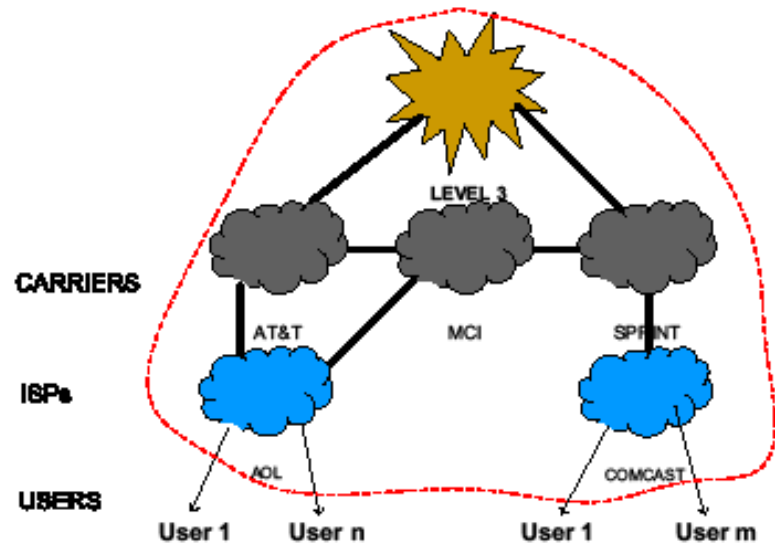


Workshop on Bandwidth on Demand 2008
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Salvador da Bahia, Brazil

Outline

- 1. Motivating Resource Allocation Problem**
2. The NSP Auction Mechanism
3. Equilibrium Properties
4. The Double-sided Version for Resource Exchange

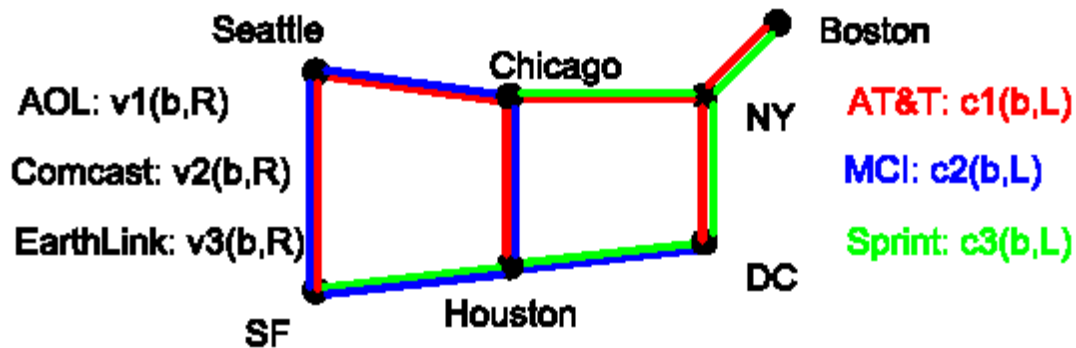
Internet Structure



- ★ *Domains*: Owned and operated by independent entities – have own information and are selfish
- ★ *Players*: Users, ISPs, Carriers
- ★ *Question?* How can we devise mechanisms such that these entities acting selfishly end up achieving the system objective

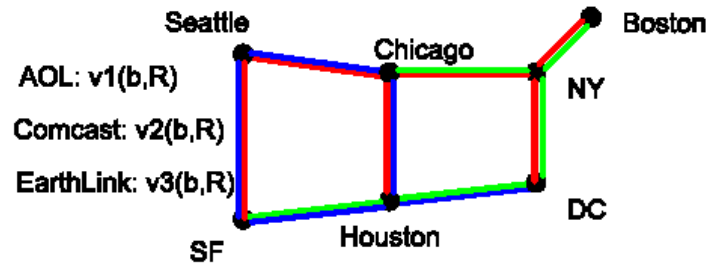
Decentralized Network Resource Exchange

- ★ Resource Allocation at the Network level:



- ★ Network operator has private cost $c_j(y; l)$ on link l
- ★ Service provider has private utility $v_i(x; R_i)$ for bundle R_i
- ★ System Objective: maximize *Social Welfare* = $\sum_i v_i(x_i) - \sum_i c_j(y_j)$

Network Resource Allocation Among Strategic Players



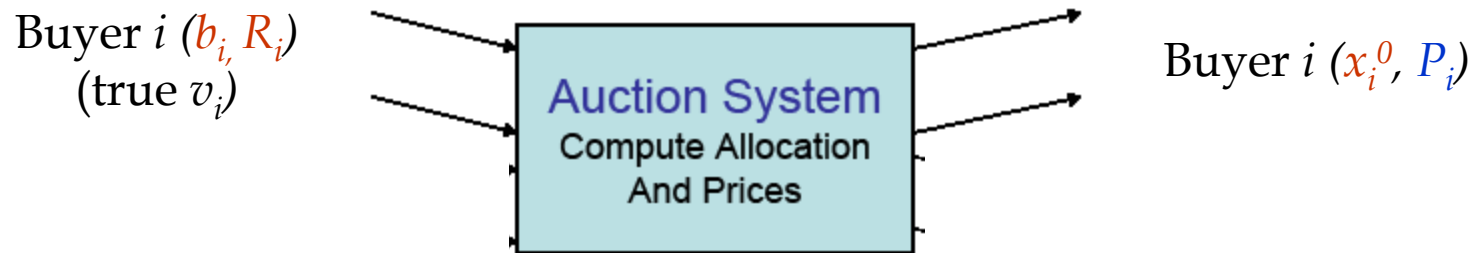
- ★ L divisible goods $1, \dots, L$; C_l units of good l
- ★ n buyers & the Auctioneer
- ★ Buyer i wants bundle R_i , has payoff function

$$u_i(x_i, w_i) = v_i(x_i) - w_i$$

- Assume: v_i is strictly increasing, concave and twice differentiable

- ★ Social Welfare: $S(x) = \sum_i v_i(x_i)$
- ★ SYSTEM OBJ: x^{**} that $\max S(x)$ sub. to capacity constraints
 - Capacity constraints: $\sum_{i \in P(l)} x_i \leq C_l \quad \forall l$
 - $P(l)$ = set of Buyers who have l in their bundle

The Auction System



- ★ Buyer i reports a bid signal b_i for bundle R_i
- ★ BUYER OBJ: To pick b_i to maximize

$$u_i(b_i, b_{-i}) = v_i(x_i(b_i, b_{-i})) - P_i(b_i, b_{-i})$$

- ★ SYSTEM and BUYER Objectives need to be aligned
 - By picking the right allocation function x^0 and payment functions P_i

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The Network Second Price Mechanism

★ Buyers submit bids $b_i = (\beta_i, d_i)$: willing to pay β_i per unit up to d_i units of bundle R_i

★ Mechanism maximizes “social welfare” sub. to capacity and demand constraints

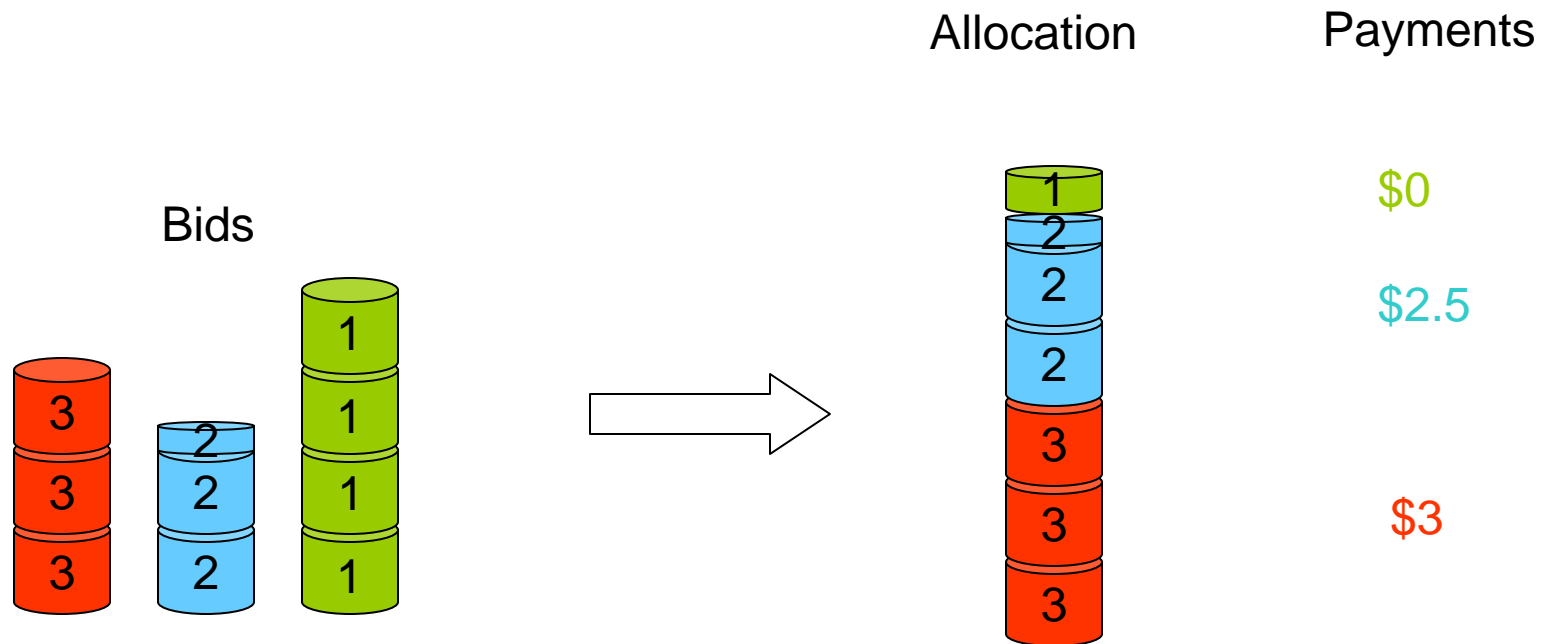
$$\begin{aligned}
 x^0 \in \arg \max \sum_i \beta_i x_i \\
 \text{s.t. } \sum_{i \in P(l)} x_i \leq C_l \quad \forall l \in [0:L] \\
 x_i \in [0, d_i], \quad \forall i=1, \dots, n
 \end{aligned}$$

★ Buyer i receives x_i^0 of R_i and pays $P_i(b_i, b_{-i}) = \sum_{j \neq i} \beta_j [x_j^0(-i) - x_j^0]$

- $x_j^0(-i)$ is auction allocation when bidder i does not participate

An Illustrative Example

- ★ **Example:** $C=6$ of one good among Three Players,
- Player **R** bids (\$3/unit, 3 units), Player **B** (\$2/unit, 2.5 units) & Player **G** (\$1/unit, 4 units)



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Nash Equilibrium Analysis

★ The payoff of buyer i is

- $u_i(b_i, b_{-i}) = v_i(x_i^0(b)) - P_i(b)$

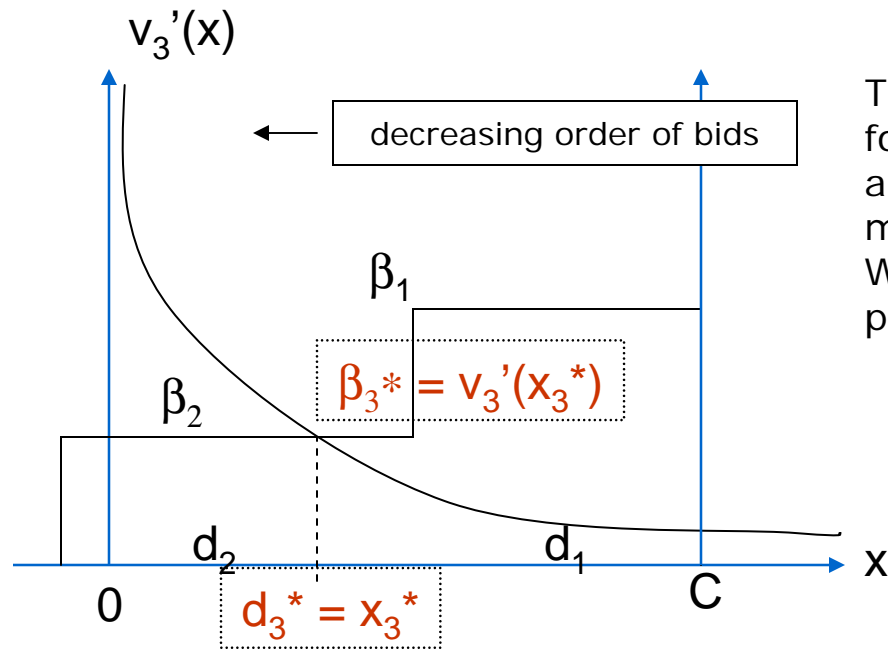
★ A Nash equilibrium is a bid profile $b^* = (b_1^*, \dots, b_n^*)$ such that

$$b_i^* \in \arg \max_{b_i} u_i(b_i, b_{-i}^*), \quad \forall i$$

★ A Nash equilibrium (allocation x^*) is efficient if $\sum_i v_i(x_i^*) = S_{max} = \sum_i v_i(x_i^{**})$

Theorem: There exists a Nash Equilibrium b^* with efficient allocation $x^* = x^{**}$

Proof Idea



The Lagrange multipliers for the auction optimization also turn out to be Lagrange multipliers for the Social Welfare maximization problem

★ Inefficiency and Reserve Prices

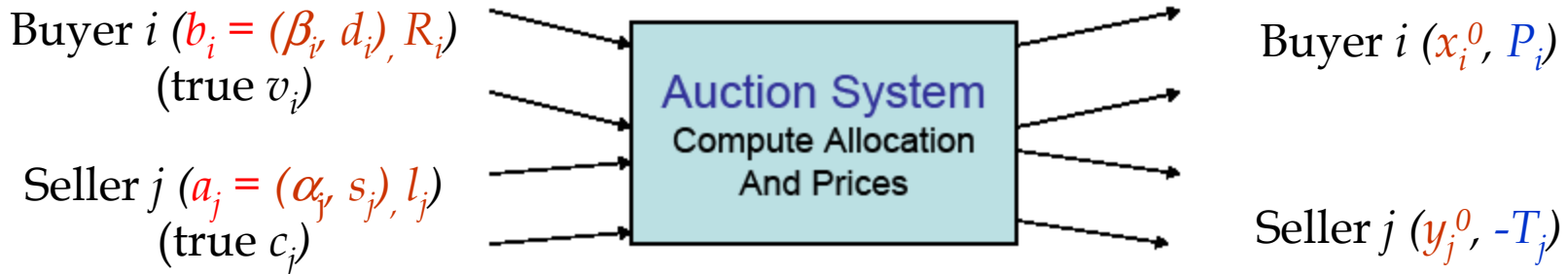
- Many Nash equilibria, not all efficient but some inefficient Nash equilibria can be eliminated through reserve prices: Each winner pays a reserve price plus usual payment

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The Double-sided NSP Mechanism

- ★ L divisible goods, n buyers, m sellers (& *The Auctioneer*)



- ★ Buyer i wants a bundle R_i , Seller j offer only one good l_j
- ★ Social Welfare: $S(x, y) = \sum_i v_i(x_i) - \sum_j c_j(y_j)$
- ★ “Capacity” constraints: $\sum_{i \in B(l)} x_i \leq \sum_{j \in S(l)} y_j, \forall l$
- ★ SYSTEM OBJ: (x^{**}, y^{**}) that $\max S(x, y)$ s.t. capacity constraint satisfied
- ★ BUYER/SELLER OBJ: Pick bids β_i, α_j to maximize their net payoffs

Theorem: There exists a Nash Equilibrium (b^*, a^*) with efficient exchange
 $(x^*, y^*) = (x^{**}, y^{**})$

Related Work

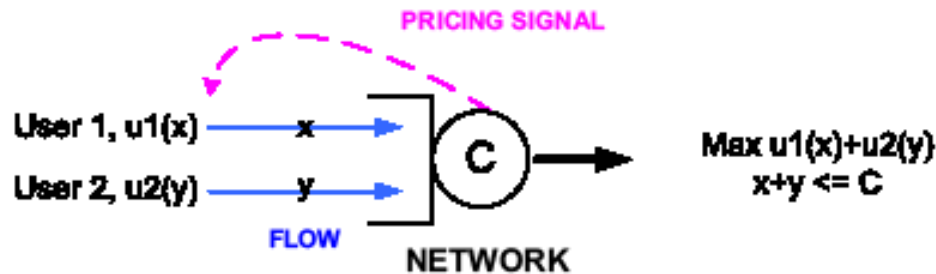
- ★ Semret and Lazar (1999) Progressive Second Price (PSP) Mechanism for a single good and only buyers
 - Maille-Tuffin (2004) provided a multi-bid generalization for single good
 - Courcoubetis, et al. (2004, 2005) suggested a PSP generalization to the network case but mechanism has efficiency problems
- ★ Johari-Tsitsiklis (2005) proposed a very general VCG-like single-sided mechanism with 1-dimensional bids and “nice” bidding functions
 - Yang-Hajek (2005, 2006) also proposed 1-dimensional bid function VCG-like mechanisms, special case
 - Maheswaran-Basar (2004) proposed a similar mechanism ESPA
- ★ Stoenescu-Ledyard (2006) give another network auction mechanism

Summary

- ★ We have proposed a VCG-like mechanism for both single-sided *and* double-sided auctions
 - Generalization of Semret and Lazar (1999) to the network case as well as double-sided case
 - Two-dimensional bids and more “natural” bidding functions
 - Generalizes to multiple routes between a source-destination pair
- ★ Showed existence of an efficient Nash equilibria
 - Some inefficient Nash equilibria can be mitigated through reserve pricing
 - Provided characterization of efficient Nash equilibria

Flow Control and Pricing

- ★ Resource Allocation at (Fine-grained) User-level:



- ★ Pricing signals can be used to decompose *Flow Optimization* problem [Kelly'97]
 - Primal-Dual decomposition: Users solve *Primal* problem, Network solves *Dual* problem
- ★ Fundamental understanding of TCP: Internet instability unlikely even with heterogeneous TCP usage [Low'99, Walrand'00, La'04, etc.]