Pricing Resources on Demand

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Access Bandwidth Markets

- Traditional contracts:
  - Buy the “right” to use a fixed amount of resources for a long period of time at a given price
- Bursty nature of demand
- Emerging technology provides mechanisms for on-demand resource allocation, e.g. “turbo” button for boosted download throughput

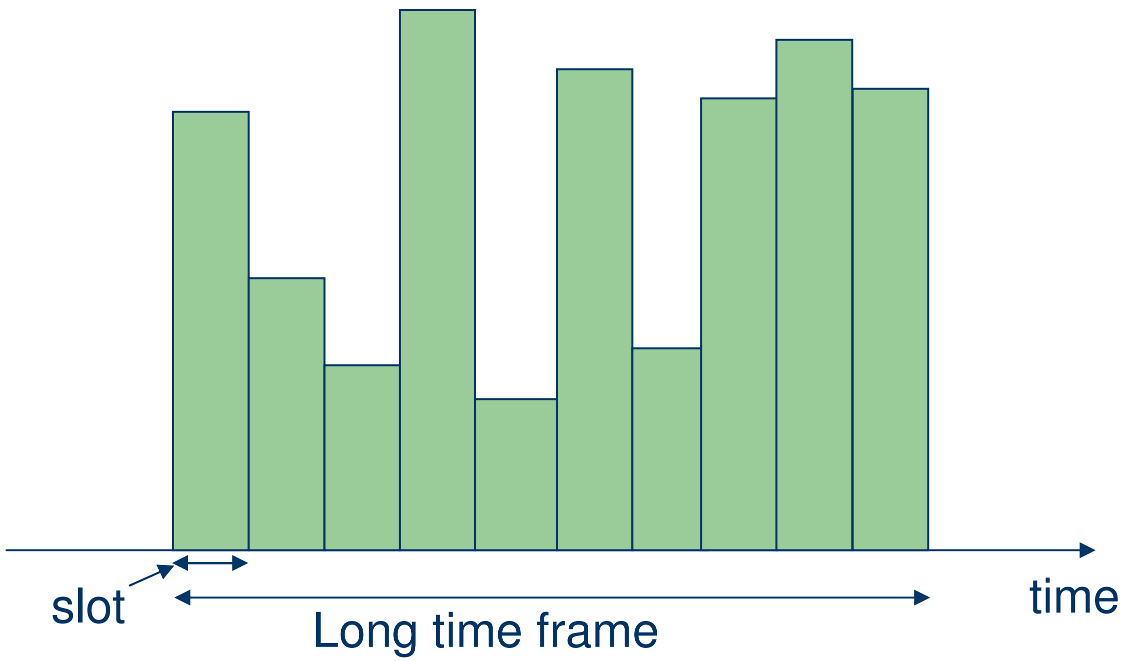
Providers & Consumers

- An important question that ISPs face is how they should price their resources, in order to maximize their profits, while keeping their customers satisfied (or better, expand their customer base).
- Consumers, on the other hand, want to have flexibility in the way they express their needs in order to maximize their net benefits.

$\Rightarrow$ New types of contracts should be made between these two parties

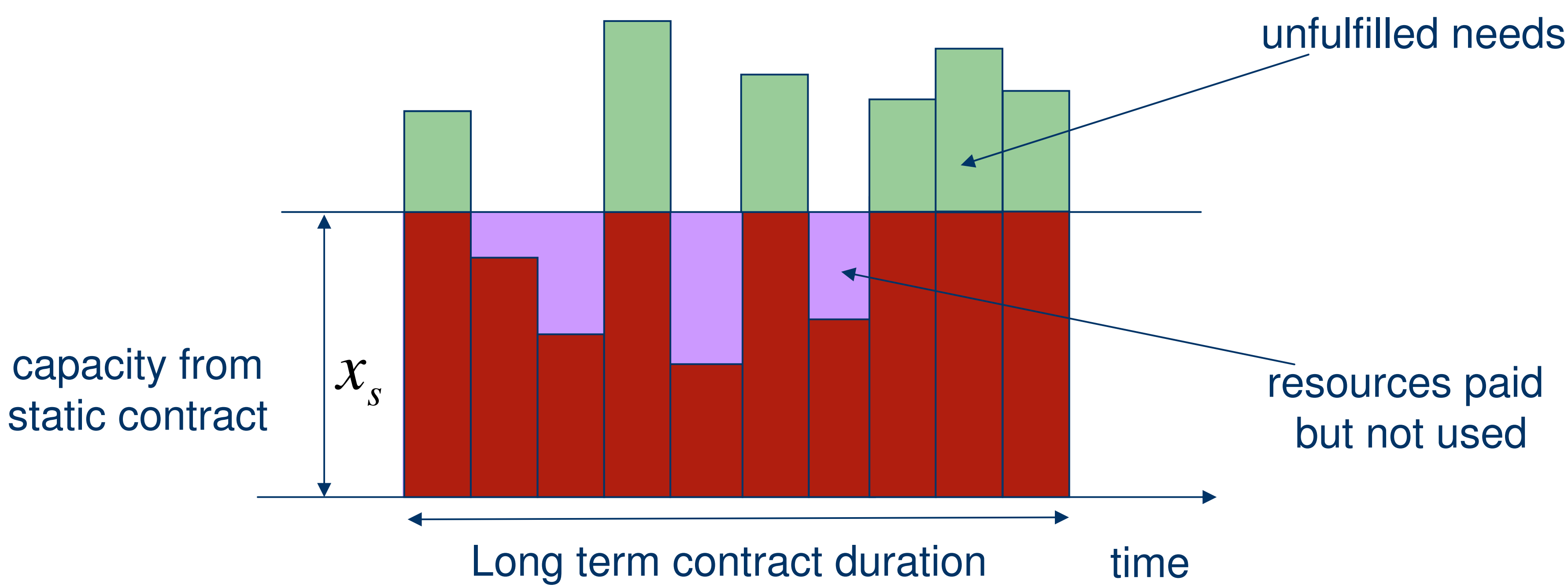
The random demand

- Demand in each time slot is random, known only at the beginning of the slot



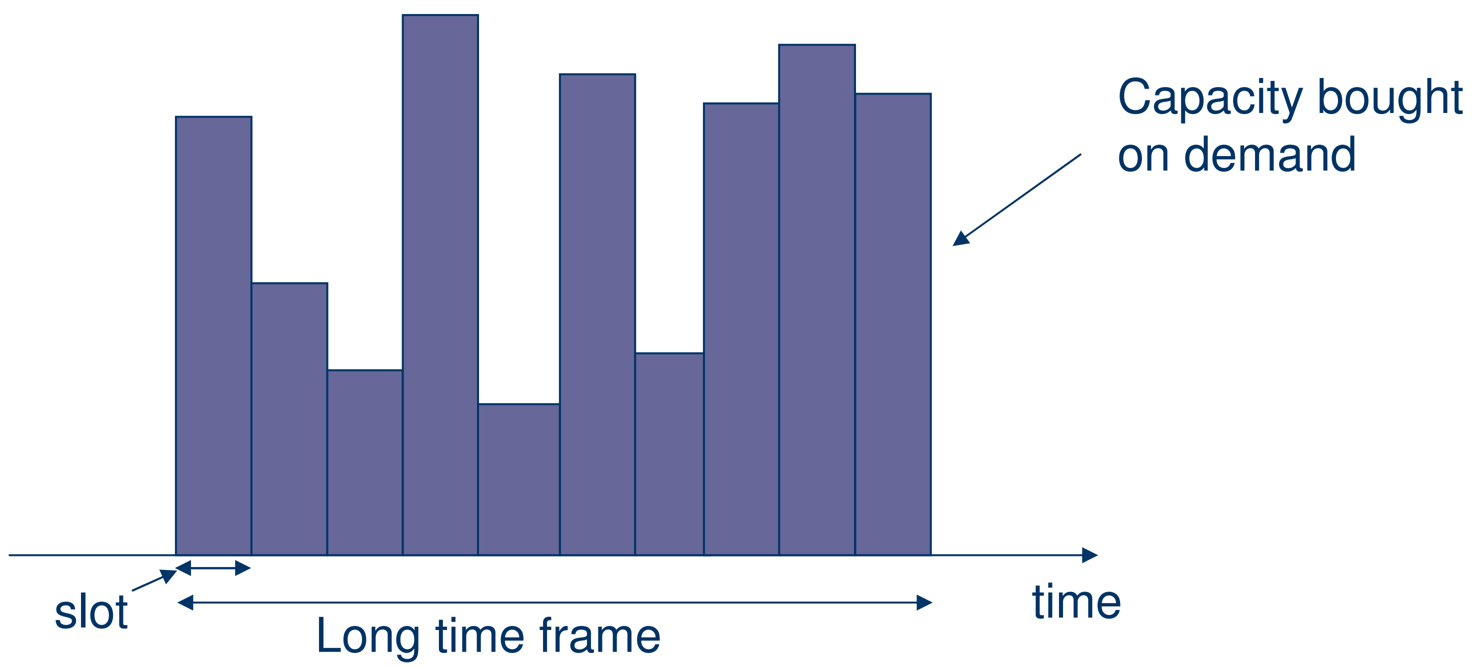
"Static" contracts

- The consumer buys <u>in advance</u> x_s units of resources at a price of "€ α_s per unit"



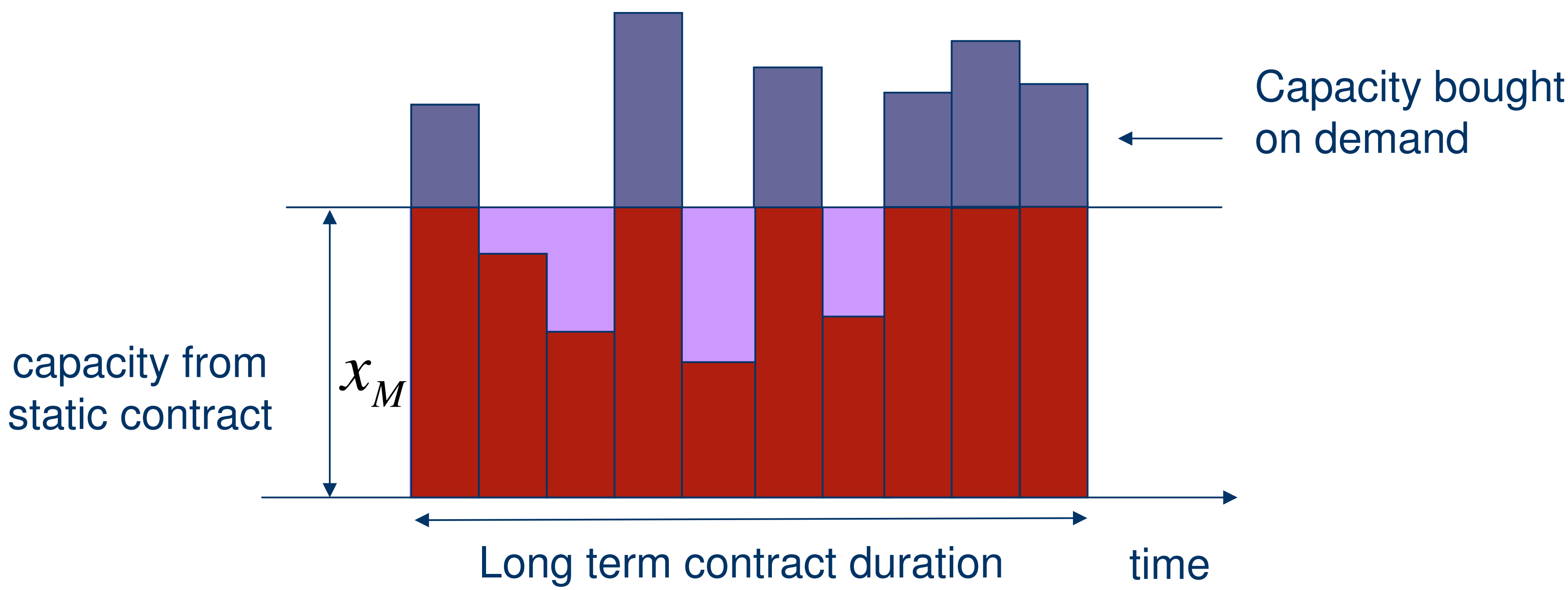
“Dynamic” contracts

- Each timeslot, the consumer buys the needed units of resources at a price of “€ b_d per unit”



“Mixed” contracts

- The consumer buys <u>in advance</u> x_M units of resources at a price of “€α_M per unit” and can also buy <u>extra resources at each timeslot</u> at a price of “€ b_M per unit”



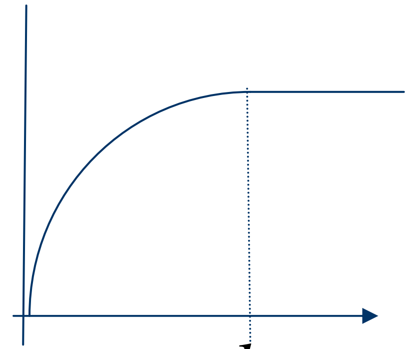
The provider

- Three types of providers
  - Static provider: offers tariff (α, ∞)
  - Dynamic provider: (∞, b)
  - Mixed provider: (α, b)
- Goal: profit maximization
  - monopoly
  - competition

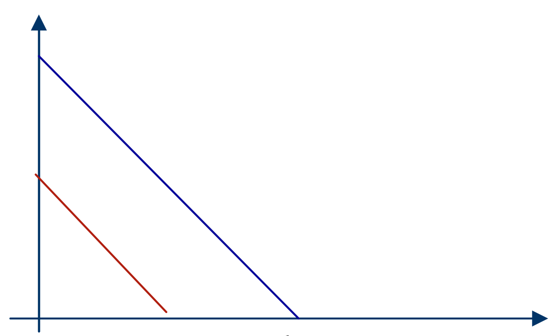
The consumer

- utility function

$$u_k(x) = \begin{cases} kx - \frac{1}{2}x^2, x \le k \\ \frac{1}{2}k^2, x \ge k \end{cases}$$



- k distributed according to $f(k)$, OR two-point distribution: $k=k_1$, with prob. p_1, $k=k_2$ with prob. p_2



The consumer’s problem

- Static contract: $nb_S(a) = \max_x \{E[u_k(x) - ax]\}$
- Dynamic contract: $nb_D(b) = E[\max_x \{u_k(x) - bx\}]$
- Mixed contract: $nb_M(a,b) = \max_x \{E[\max_y \{u_k(x+y) - by\} - ax]\}$
- Under the same prices, i.e. $\alpha = b$, dynamic contracts are more beneficial than static ones since

$$\max_x \{E[u_k(x) - ax]\} \le E[\max_x \{u_k(x) - bx]\}$$

Monopoly

Solve case where k is uniform on [0,1]

	Static (S)	Mixed (M)	Dynamic (D)
Seller revenue	0.0741	0.0800	0.0741
Optimal α	0.2222	0.2400	
Optimal b		0.4000	0.3333
BW bought in static	0.3333	0.2000	
Mean BW bought	0.3333	0.2800	0.2222
User net benefit	0.0432	0.0440	0.0494

Initial conjectures:

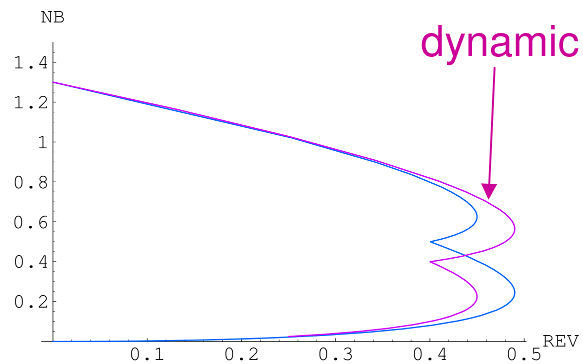
A. The revenue achieved by the seller is strictly greater in (M) than in (S) or (D).
B. The mean bandwidth sold decreases from (S) to (M) to (D).
C. The user's average net benefit increases from (S) to (M) to (D).

Some interesting results (1)

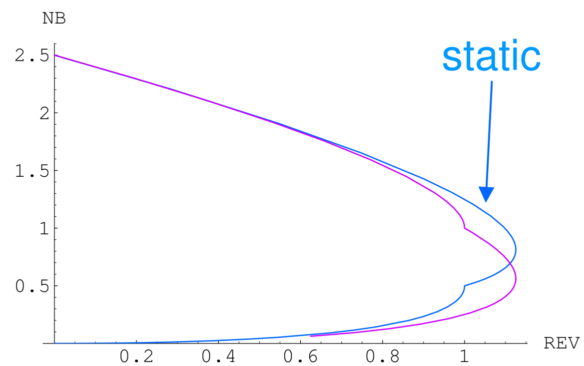
- Revenue equivalence theorem: if k is arbitrarily distributed on [0,1] then a provider of static contracts achieves the same maximum revenue with a provider of dynamic contracts
- At these optimums, it stands that $x_S{}^* = b_D{}^*$

Some interesting results (2)

- If k is two-point distributed and $p_1 = 1 - p_2$, the condition for a customer to strictly prefer a provider of static contracts over a provider of dynamic contracts is $$p_2 > p^* = \frac{1}{(k_2 / k_1 - 1)^2}$$
  The preference is reversed when $p_2 < p^*$
- Note that: $2\,k_1 \geq k_2 \Rightarrow p^* \geq 1 \Rightarrow p_2 \leq p^*$



p_2=0.2



p_2=0.5

k_1=1, k_2=3, p^*=0.25

Summary of monopoly results

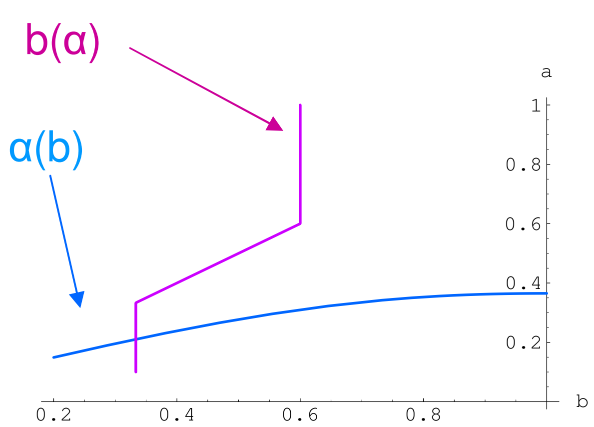
- Summarizing the results of the two-point distributed k model

$p_2 \leq p^*$	$p_2 > p^*$
$REV_S = REV_D < REV_M$	$REV_S = REV_D < REV_M$
$NB_S < NB_M < NB_D$	$NB_D < NB_S < NB_M$
$AVBW_S = AVBW_M = AVBW_D$	$AVBW_S = AVBW_D > AVBW_M$

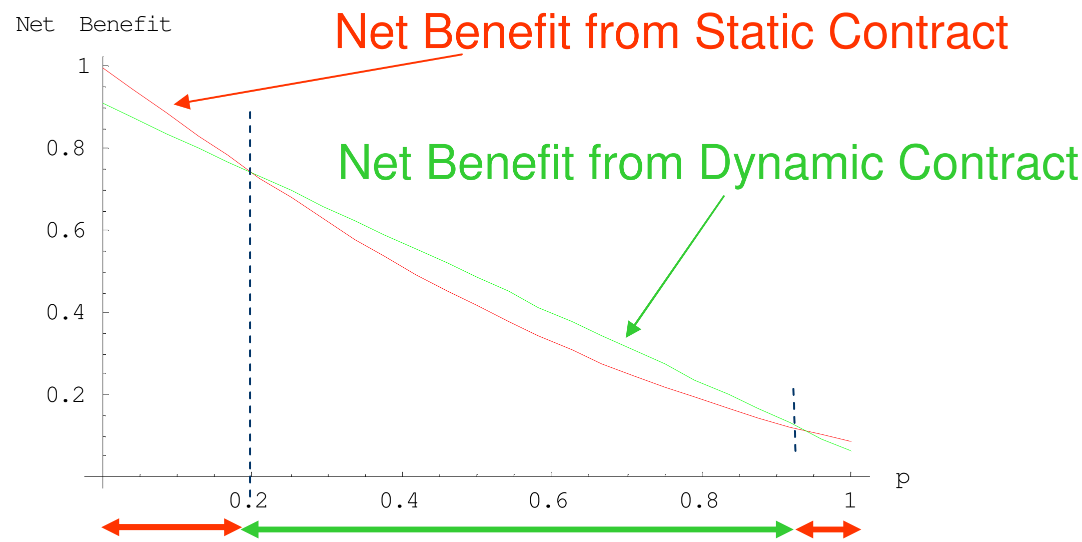
- Mixed contracts are always preferable to static ones, in the sense that
  - $REV_M > REV_S$
  - $NB_M > NB_S$
  - $AVBW_M \leq AVBW_S$

Competition: static vs dynamic

- A static provider competes in prices with a dynamic provider
  - In traditional Bertrand competition: the provider with the smaller cost will win
  - in our case: both providers make money in the equilibrium



Reaction curves



Market share

k_1=0, k_2=1, c_S=0.1, c_D=0.2, p: uniformly distributed in [0,1]

Thank you!

Questions?